DOES GROWTH MATTER FOR CYCLES?

PEDRO SERÔDIO

Department of Economics, University of Warwick

ABSTRACT. The relatively successful consolidation of dynamic stochastic general equilibrium models as the preferred modelling strategy in macroeconomics has had, so far, largely abstracted from the problem of how productivity growth is generated and instead focused on refining its performance around a stationary steady-state. Incorporating endogenously determined productivity growth is likely to affect the internal propagation mechanism of a standard RBC model through a feedback between output and the growth rate and, if so, abstracting from that process may lead to the model underreporting output fluctuations both at higher and lower frequencies. I develop three alternative specifications for the innovation generating research sector and compared the performance of those models with a benchmark RBC model with exogenous growth. All alternative specifications fail to outperform the standard RBC model at higher frequencies and only modestly improve over lower frequencies, but do reasonably well at capturing the behaviour of R&D over the cycle at all frequencies.

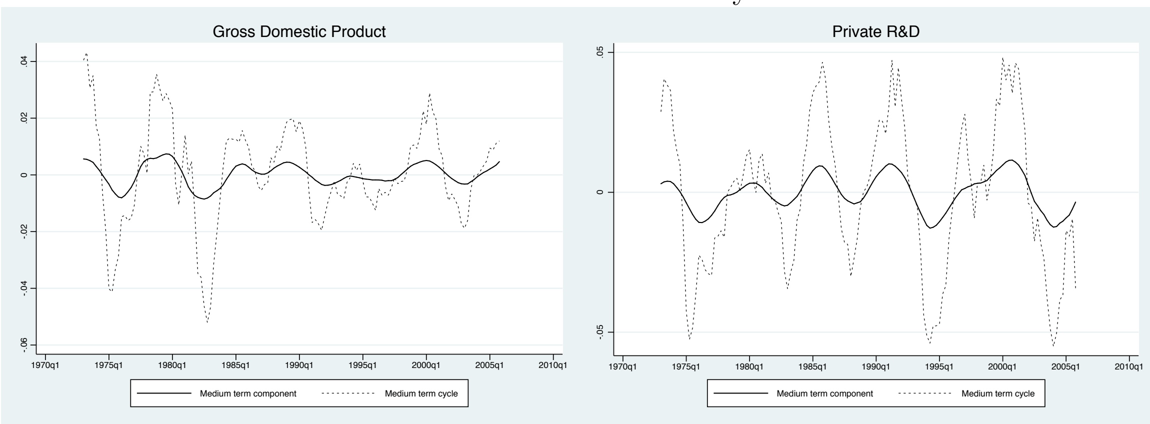
1. Introduction

Endogenous growth theory and dynamic stochastic general equilibrium modelling have until relatively recently followed divergent paths, with the former concerning itself exclusively with the driving forces of long term productivity growth and the other with the cyclical behaviour of economies around this long term trend. A consequence of the lack of overlap between these approaches concerns the disconnect between the long run trend and short run fluctuations in key macroeconomic variables, which could have implications for the analysis of both higher and lower frequency fluctuations by ignoring possible feedback mechanisms between output and output growth.

Although there have been few contact points between the two approaches, the relationship between cycles and growth has always been an important question in the endogenous growth literature, with the twin issues of whether cycles have a significant impact on long run productivity growth or whether growth itself is the result of disruptive innovations that generate short term fluctuations featuring prominently. The lack of a unified approach or modelling strategy and the great diversity of plausible growth generating mechanisms has meant that until very recently that gap has remained wider than would be expected given the relative importance of the two research areas and the potential benefits to be gained by adopting a common language and methodological approach largely unrealised.

Furthermore, as argued by Comin and Gertler (2006), focusing exclusively either on short run fluctuations or long-run balanced growth paths ignores an important dimension of the business cycle, namely, the existence of lower frequency cycles that can be easily identified in US data which may be tied to short run fluctuations through the process of generating long run growth productivity growth. Integrating both approaches may be important for the study of the business cycle if short run fluctuations are partly the result of longer cycles at lower frequencies which are ignored by current approaches. Figure (1) shows two measures of these lower frequency cycles, with the dotted lines being the result of filtering the data with a bandpass filter for a frequency between 2 and 200 quarters, while the solid line is the result of frequencies between 32 and 200 quarters. Subtracting the latter from the former yields the higher frequencies that are standard in the literature and comprise the vast majority of the analysis. Both graphs highlight the fact that despite being a much lower amplitude, there is a substantial degree of variation that is not captured by higher frequency filters.

FIGURE 1. Medium term cycles



More recently, some significant contributions attempting to bridge the gap between the two strands of the literature by embedding the technological progress generating mechanism in business cycle models have been proposed, but they have either been relied on learning by doing mechanisms or built on the idea of technology expanding the set of producible goods, product or ideas. Models built in the latter tradition are often described as models of 'expanding varieties'. In contrast, very few contributions attempting to incorporate the common 'quality ladder' framework in a standard business cycle model have been proposed so far.

In this paper I describe how a simple quality ladder model for labour augmenting technological progress can be easily embedded in a standard real business cycle model, and propose two simple extensions to allow both entrants and incumbents to engage in innovative activity and contribute to productivity growth, as well as endogenously determined mark-ups and market structure through technology imitation. The simplest of these is based on the Schumpeterian growth model described in Açemoglu (2007) and outlines the basic creative destruction process that underlies all these models. It can be described as follows: incumbents hold a patent for production of the sectoral good at the current quality level. Challengers engage in research so as to improve upon the quality level which, if successful, allows them to acquire a patent and therefore dislodge the incumbent, who exits the market. The Arrow effect prevents incumbents from engaging in research effort[1], which means all growth comes from innovators trying to usurp established incumbents.

The first of these extensions is a variation of the model discussed in Açemoglu and Cao (2010), which allows for innovative activity by both incumbents and entrants and arises because entrepreneurs may face very different incentives to engage in research; being locked out of the market, they do not face a trade-off between production and research expenditure and, therefore, the behaviour of research spending when only entrepreneurs are allowed to innovate may differ substantially from what undoubtedly occurs in settings where companies face a trade-off between production and investment in R&D. The mechanisms discussed previously still present themselves, in that entrants will still displace incumbents if successful in their development of improvements to the current quality level of the product. The main departure from the standard framework is that the probabilities of innovating successfully are different for challengers and current producers. The process of creative destruction is still very much present but established firms are now able to retain their market position by innovating, and, therefore securing a patent which shuts potential rivals out of the market for its duration (at least one period and for as long as no challenger successfully innovates). In calibrating the model, I chose parameter values which reflect two intuitive propositions about the innovative process in incumbents and challengers; the former are assumed to be much more likely to be successful in developing higher quality versions of the product while entrants, when successful, are assumed to discover more disruptive, or radical, innovations which imply larger jumps to the overall quality level.

The second extension concerns the role of market structure and how this may affect the internal propagation mechanism of the model. Following Galí and Zilibotti (1995), I derive a mark-up that is endogenously determined by allowing established firms to copy newly developed technologies. This generates a process of entry and exit by removing

[1]They will always be outbid because the potential gain to entrants is always higher.

patents and allowing the price mark-up charged by incumbents to be a function of the number of firms in the market. Agents are assumed to be equipped to produce only if they possess the relevant production technology, which they can acquire either by improving upon the existing quality level, i.e., innovating, or, if they are incumbents, by attempting to copy new innovations as they become available. Thus, the market structure at any point in time is comprised of a successful innovator, an entrant, and previous incumbents that successfully adopted a version of the technology and therefore remained relevant in the sector [2]. Thus, the number of firms and the price mark-up are co-determined endogenously by the processes of innovation and imitation, implying that the market structure in the relevant sector is entirely flexible and, consequently, that these variables respond to cyclical fluctuations, potentially amplifying their impact on the response of output to exogenous shocks..

All models are calibrated according to standard values in the DSGE literature and the relevant variables to the model are matched with first moments observed in US time series. Doing this enables comparisons of the behaviour of key variables across all three models and against second moments extracted from the data. I then compare their performance with a benchmark RBC model.

2. Literature Review

As early as King and Rebelo (1986), authors have suggested that the assumption of independence between the growth generating processes and output fluctuations in the short term may be unjustified once the relationship between growth and cycles is taken seriously. In particular, the authors argue that including a growth generating mechanism in a standard real business cycle model imply a substantially different performance of the model in terms of the response of key variables to exogenous shocks.

One popular approach to incorporating endogenous technical change relies on processes of learning by doing or human capital accumulation as the engines of growth. Among these is the seminal contribution of Stadler (1990), who equally suggests that once these

[2] Entrants are assumed not to be able to copy existing technology because they do not possess it.

growth generating mechanisms are duly accounted for in the model, its behaviour is substantially different to that of a model in which the growth components are orthogonal to the model's propagation mechanisms. Further contributions by Ozlu (1996), Pelloni (1997) and Moral Zuazo and Barañano (2003), report an improvement in a standard RBC model's ability to match key moments in the data when technological progress, of the learning by doing or human capital accumulation variety, is explicitly accounted for. Jones et al. (2000) on the other hand argue that while shocks have some impact on endogenous growth rates, these effects are quantitatively small for appropriate calibrations.

In a model in which innovations arrive stochastically as the result of human capital accumulation, Maliar and Maliar (2004) show that for larger ratios of human to physical capital, the propagation mechanism of a standard RBC model is improved and the behaviour of output and output growth more closely matches empirical observation. Learning by doing and human capital accumulation models, however, have the undesirably property of implying that the input to the innovation process behaves counter-cyclically, which is at odds with findings that research expenditure is, at least, moderately pro-cyclical.

Alternative specifications either take the form of expanding variety models in the spirit of Comin and Gertler (2006), Comin et al. (2009) and Schmid and Kung (2011) or the so-called Schumpeterian models of quality ladders. This paper follows the latter approach so as to capture the potentially disruptive effect of creative destruction, but borrows liberally from Comin and Gertler (2006), approach to defining the medium-term business cycle.

A contribution that closely follows the approach used in this paper is that of Phillips and Wrase (2006), who, despite not significantly improving on the performance of the standard RBC model in terms if matching relevant second moments, come close to mimicking important features of the data. They do so despite no labour-leisure trade-off and with a very simple description of all non-Schumpeterian aspects of the economy. This suggests that, with appropriate extensions and a richer model namely one in which there is a more complex mechanism of technology diffusion this approach has significant potential.

While some authors have argued that incorporating endogenous growth through learning by doing or capital accumulation noticeably improves the propagation mechanism of a standard RBC model, reliance on labour as the sole input for the innovative process is problematic because of its counter-cyclical nature. I propose a version of the Schumpeterian quality ladder model in which the final good, rather than labour, is used as the input for the innovation process and explore how accounting for growth explicitly significantly changes the way a simple real business cycle model matches key moments in the data.

3. Model

All of the models discussed in this section share a few common features: they include endogenous technology adoption, which evolves according to improvements to quality rather than the introduction of new varieties[3]; a standard real business cycle model with shocks to aggregate productivity and no frictions in product, labour or financial markets. I start by discussing the elements which are common to all the versions discussed throughout the paper. The most significant of these is the consumer problem, which is standard in the literature.

3.0.1. *Households.* The representative consumer in this economy must decide on the optimal amounts of consumption of the final good as well as the amount of labour it supplies each period, and the optimisation problem takes the following form:

$$\max_{\{c_t(\iota),h_t(\iota),B_t(\iota),k_t(\iota)\}_{t=0}^{\infty}} U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log c_t(\iota) - \varphi \frac{h_t(\iota)^{1+\psi}}{1+\psi} \right\} \tag{1}$$

$$\text{s.t.} \quad w_t h_t(\iota) + s(\iota) \int_0^1 \pi_t(e)de + R_t^k k_t(\iota) + R_t^b B_t(\iota) =$$

$$= c_t(\iota) + k_{t+1}(\iota) + B_{t+1}(\iota) + s(\iota) \int_0^1 \eta(z_{t-1}^E(e)) V_t^j(q_i(e))de \tag{2}$$

The representative consumer's earnings take the form of a salary derived from providing labour services to sectoral goods producers ($w_t h_t$), profits generated by these same producers (π_t), gross interest on rental of units of capital (R_t^k) and gross interest on loans made to entrepreneurs ($R_t - 1$). Final consumers then spend their income on consumption of the final good, purchases on new capital goods, new loans to entrepreneurs and

[3]Hence the Schumpeterian element. Creative destruction, i.e., the replacement of incumbents by entrants is crucial to drive the innovation process.

purchases of successful new companies. This last expenditure item reflects the structure of firm ownership in this environment and would make no difference in equilibrium to allow entrepreneurs to possess the rights to the flow of profits instead. The entrepreneur's optimisation problem can be similarly defined:

$$
(3) \qquad \max_{\{c_t^E(e), L_t(e), z_t^E(e)\}_{t=0}^{\infty}} U^e = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log c_t^E(e) \right\}
$$

$$
\text{s.t.} \qquad c_t^E(e) + z_t^E(e) q_i(e) + R_t^b L_t(e) = L_{t+1}(e) + \eta(z_{t-1}^E(e)) V_t^j(q_i(e))
$$

$$
R_{t+1}^b L_{t+1}(e) \leq \eta(z_t^E(e)) \mathbb{E}_t[V_{t+1}^j(q_{i+1}(e))]
$$

In the absence of wage earnings, entrepreneurs rely exclusively on loans from the wage earning households and windfalls from the sale of successful innovation embodying firms. The budget constraint for the representative entrepreneur states simply that the inflow of resources into the household in the form of loans and sales of newly established firms must be split between consumption, research expenditure and debt plus interest repayments. The second constraint imposes that the total amount lent must not exceed the expected return from investing in research, taking into consideration the opportunity cost of the wage earning households in the form of the interest rate on loans.

Combining the first order conditions for both types of household implies that the rate of return on capital must be equal to the rate of return on loans to the entrepreneurial household, which in turn, via the last constraint on the optimisation problem for entrepreneurs, must equal the internal rate of return of investing on research.

The mechanics of how research and development generates growth in aggregate productivity will be explored in further sections, but decisions over optimal investment affect the budget constraints of both types of household and must, therefore, be introduced before the innovation process is described. It is standard in the growth literature[4] to describe the entrepreneur's problem as a dynamic programming problem in which these agents choose the amount of research expenditure that maximises their net worth for a given expected

[4]See Barrau (2010), for example.

value of successfully innovating:

$$V_t^E(e) = \max_{z_t^E(e)} \left\{ -z_t^E(e) q_i(e) + \eta(z_t^E(e)) \mathbb{E}_t \left[\frac{V_{t+1}^j(q_{i+1}(e))}{R_t} \right] \right\}$$

A free entry condition is then imposed so that the net value of an entrepreneur is zero $V_t^E(e) = 0$. This, however, is only feasible if the function for the probability of successfully innovating, $\eta(\cdot)$, is linear in z_t^E. When it is not, the expected value of being an entrepreneur cannot be zero. By modelling entrepreneurs as a single agent in each sector that uses this surplus as consumption, this potential complication is avoided.

3.0.2. *Capital goods production.* Capital used in the production of intermediate goods is produced and supplied competitively, making use of a linear technology that transforms undepreciated capital purchased from wage-earning households into new capital that is then sold to these households. Absent adjustment costs, the cost of a unit of capital is the same as the price of final consumer good which, being the numéraire in this economy, is equal to unity. The aggregate capital stock evolves according to:

$$K_{t+1} = (1 - \delta) K_t + I_t \tag{4}$$

3.0.3. *Resource constraint.* Combining the budget constraints for both types of household, we arrive at the resource constraint for this economy:

$$Y_t = C_t + C_t^E + I_t + Z_t \tag{5}$$

Where the total amount of research expenditure, z_t, will vary according to the variant of the model being analysed.

3.1. **Standard quality ladder model.** In this section I describe the main constituent parts of a simple quality ladder model. Key features like the production of the final good or the structure of the quality ladders change between all of the versions explored here, so I will introduce them individually and discuss the implications of each.

3.1.1. *Final good production.* There is a single, perfectly competitive final producer that purchases all the sectoral goods and aggregates them using the following aggregation technology:

$$Y_t = \left[\int_0^1 \left(u_t \omega_t(e) y_t(e)^{\frac{\mu-1}{\mu}} \right) de \right]^{\frac{\mu}{\mu-1}} \tag{6}$$

Where $\omega(e)$ is the quality adjustment for sectoral output in the final good aggregator that ensures that aggregate productivity is labour augmenting:

$$\omega_t(e) = \frac{q_t(e)}{\left(\int_0^1 q_t(e)de\right)^{\frac{1+\alpha(\mu-1)}{\mu}}} \tag{7}$$

This ensures that in a symmetric equilibrium in which sectoral output is identical across all sectors, final output is linear in average quality. The elasticity of substitution between sectoral goods, μ, follows directly from the Dixit-Stiglitz aggregator and is the mark-up over marginal cost in equilibrium. The neutral aggregate productivity shock u_t obeys the following law of motion:

$$u_t = \rho_u u_{t-1} + \epsilon_t^u \tag{8}$$

Each sector is assumed to have a single producer of the intermediate good who does so with the following production function:

$$y_t(e) = [k_t(e)]^\alpha [h_t(e)]^{1-\alpha} \tag{9}$$

The intermediate good producer must then decide on an optimal pricing strategy. While she possesses a patent for the most recently developed technology, the mark-up over marginal cost must be set to ensure that previously displaced incumbents cannot compete by reducing their profits. Jumps in the quality ladder, which will be defined ahead, must then be large enough so as to preclude that possibility. In particular, the following condition must hold:

$$\lambda^{1-\alpha} > \mu$$

As we shall see, each step in the ladder is λ times more productive than the former and, working through (7), a displaced incumbent has a marginal cost that is exactly $\lambda^{1-\alpha}$ higher than that of an entrant in possession of a more efficient technology. When innovations are "drastic"enough, i.e., the quality jump is sufficiently large, the proprietor of the most recent technology can charge the unconstrained monopoly price. This can be computed by taking the demand for each sectoral good:

$$p_t(e) = \left(\frac{y_t}{y_t(e)}\right)^{\frac{1}{\mu}} w_t(e)$$

And then solving the first of two optimisation problems. The first pertains to the choice of an optimal mark-up over marginal cost:

$$\max_{y_t(e)} \pi_t(e) = (p_t(e) - mc_t(e))y_t(e)$$

And the second the optimal choice of inputs for a given marginal cost:

$$\min_{k_t(e),h_t(e)} (R_t - 1 + \delta)k_t(e) + w_t h_t(e)$$
$$\text{s.t.} \qquad mc_t(e)y_t(e)$$

Solving for $y_t(e)$, $k_t(e)$ and $h_t(e)$ gives the monopolistic mark-up over marginal cost, the demand for capital goods and the demand for labour services:

$$p_t(e) = \mu mc_t(e) \tag{10}$$

$$\alpha y_t(e) = \mu(R_t^k - 1 + \delta)k_t(e) \tag{11}$$

$$(1 - \alpha)y_t(e) = \mu w_t h_t(e) \tag{12}$$

Finally, replacing (10) into the monopolist's optimal pricing decision, we get the following profit function:

$$\pi_t(m) = \frac{\mu}{\mu - 1} p_t(e)y_t(e) \tag{13}$$

3.1.2. *Technology and research.* The level of quality in each sector evolves according to the following ladder structure:

$$q_{i+1} = \lambda q_i$$

In which, following each successful innovation, moving from one step to the following increases that sector's quality level by $\lambda - 1$. This means that for a given initial quality level q_0, the i-th level of quality is $\lambda^i q_0$. As described in their optimisation problem, entrepreneurs invest an amount $z_t^E(e)q_t(e)$ in research and development. Total expenditure is a function of the quality level, to account for the increasing difficulty of conducting research as each sector becomes more technologically sophisticated. This is made clearer in the probability of successfully coming upon an improvement to the quality level, which is then a function of the quality adjusted level of expenditure:

$$\Pr(\text{entrepreneur } e \text{ is successful}) = \eta(z_t^E(e)) = \eta(z_t^E(e))^\gamma, \qquad 0 < \gamma < 1, \quad \eta > 0$$

A more complete justification for this specific functional form can be found in Açemoglu and Cao (2010), who outline the argument for $\eta(\cdot)$ displaying decreasing returns to scale in the total amount of research and development undergone by potential entrants[5].

From the first order conditions to the entrepreneur's problem in (3), the optimal research intensity is given by:

$$q_i(e) = \frac{d\eta(z_t^E(e))}{dz_t^E(e)} \mathbb{E}_t \left[\frac{V_{t+1}^j(q_{i+1}(e))}{R_t^b} \right] \tag{14}$$

In short, the entrepreneur optimally chooses the level of research intensity $z_t^E(e)$ and, if successful in developing a new product, the incumbent loses its monopoly position in the sector and is replaced by a new firm which is then sold to the wage-earning household. If the research program does not yield a discovery, then the entrepreneur gains nothing and the incumbent retains the monopoly position. The value of this position, i.e., the value of a company, at time t reflects the per period flow of resources, which are the firm's operational profits $\pi_t(e)$ and the discounted continuation value, which is a function of the probability of retaining a position of incumbency. This is simply:

$$V_t^j(q_i(e)) = \pi_t^j(e) + \left(1 - \eta(z_t^E(e))\right) \mathbb{E}_t[\Lambda_{t,t+1}(\iota) V_{t+1}^j(q_i(e))] \tag{15}$$

The last term on the right hand side of the equation represents the probability of no innovation happening in sector e with the discounted continuation value of the firm in the next period. Because all companies are ultimately held by the wage-earning households, this value is discounted using the representative household's stochastic discount factor[6], defined as:

$$\Lambda_{t,t+1}(\iota) = \mathbb{E}_t \left[\beta \frac{c_t(\iota)}{c_{t+1}(\iota)} \right] \tag{16}$$

[5]This can be interpreted as there being excessive competition for ideas. Should there be an increase in aggregate spending in research and development by entrants, good ideas will be harder to come by per unit of expenditure. Açemoglu and Cao (2010), describe this phenomenon as "fishing out of the same pond".

[6]Equilibrium requires that the returns on all possible investments - capital, loans and investment and research and development - earn the same rate of return and, therefore, the ownership structure is immaterial in this context.

Incumbents can only affect the per period flow of profits through decisions on capital and labour use but optimally opt out of the innovation market. This is simply the well established Arrow replacement effect, which implies that an incumbent will always be priced out of the market for new innovations because she will always earn a lower marginal gain per each unit of investment than the entrepreneur. This is easy to by equating marginal cost and benefit for any given incumbent; for a per unit cost of doing research $q_i(e)$, we have the following equilibrium condition:

$$q_i(e) = \frac{d\eta(z_t^j(e))}{dz_t^j(e)}\mathbb{E}_t\left[\frac{V_{t+1}^j(q_{i+1}(e)) - V_{t+1}^j(q_i(e))}{R_t^b}\right] \leq \frac{d\eta(z_t^E(e))}{dz_t^j(e)}\mathbb{E}_t\left[\frac{V_{t+1}^j(q_{i+1}(e))}{R_t^b}\right]$$

For a positive firm value at time t, this implies that as the entrepreneur's marginal gain is always higher, she will always outbid the incumbent for the same research intensity. This, of course, critically relies on the assumption that incumbents do not have access to better innovation generating technologies, a rather stringent condition that will be relaxed in subsequent versions of the model.

3.1.3. *Aggregation and equilibrium.* Having fully defined all the relationships at the sector level, and alluding to the two aggregate relationships defined in (4) and (5), it is then straightforward to defined all the relevant variables at the aggregate level. The first step involves defining the average quality level, a useful construct that will allow us to express all the non-stationary variables in the model as the product of a non-stationary and a stationary quantities. This average quality level is defined as:

$$Q_t \equiv \int_0^1 q_t(e)de \tag{17}$$

The sectoral quality level changes whenever an entrepreneur successfully develops an improvement to the existing technology, which at the aggregate level implies that changes to average quality are given by simply taking the expectation over the measure of sectors. A few assumptions underpin the law of motion for aggregate quality: patent race ties are ruled out by forcing the measure of entrepreneurs to equal the measure of sectors, implying a one-to-one relationships between the two[7], probabilities of innovation across sectors are independent[8] and the probability of more than one innovation per period if

[7]This means two entrepreneurs, n and m, could both innovate simultaneously with a small probability. By assuming there is a 'single' entrepreneur in each sector, that possibility is ruled out.

[8]So that $\eta(n) \cdot \eta(m) = 0$.

vanishingly small. Under these assumption, average quality obeys the following law of motion:

$$\mathbb{E}_t\left[Q_{t+1}\right] \approx \left(\int_0^1 \eta(z_t^E(e))q_t(e)de\right)\lambda + \left(\int_0^1 q_t(e)de - \int_0^1 \eta(z_t^E(e))q_t(e)de\right)$$
$$= \left(\int_0^1 \eta(z_t^E(e))de\right)\lambda Q_t + \left(1 - \int_0^1 \eta(z_t^e(m))de\right)Q_t$$

The expected growth rate for the average quality level follows immediately from the equation above:

$$\frac{\mathbb{E}_t\left[Q_{t+1}\right] - Q_t}{Q_t} = \mathbb{E}_t(g_{t,t+1}) \approx (\lambda - 1)\int_0^1 \eta(z_t^E(e))de \tag{18}$$

Assuming that output is the same across all sectors, the production function for the economy collapses to:

$$Y_t = u_t Q_t^{1-\alpha} y_t(e) = u_t Q_t^{1-\alpha}[k_t(e)]^\alpha [h_t(e)]^{1-\alpha} = u_t k_t^\alpha \left[Q_t h_t\right]^{1-\alpha}$$

Aggregation of all the other variables follows straightforwardly, with $C_t = \int_0^1 c_t(\iota)d\iota$, $s = \int_0^1 s(\iota)d\iota = 1$, $h_t = \int_0^1 h_t(\iota)d\iota = \int_0^1 h_t(e)de$, $\Pi_t = \int_0^1 \pi_t(e)de$, $K_t = \int_0^1 k_t(\iota)d\iota = \int_0^1 k_t(e)de$, $L_t = \int_0^1 B_t(\iota)d\iota = \int_0^1 L_t(e)de$, $\eta(z_t^E) = \int_0^1 \eta(z_t^E)de$, $V_t = \int_0^1 V_t(e)de$, $C_t^E = \int_0^1 c_t^E(e)de$, $Z_t = z_t^E Q_t = \int_0^1 z_t^E(e)q_t(e)de$.

The resource constraint implies that all expenditure variables must grow at the rate of growth of the average quality level, which allows us to redefine all the variables in the model in terms of their stationary counterparts though the following simple transformation:

$$\tilde{x}_t = \frac{X_t}{Q_t}$$

Using that transformation, the equilibrium conditions for the entire model can be rewritten in terms of stationary variables, which can be analysed using standard tools just like any RBC model. Additionally, because the equilibrium expenditure in R&D is already independent of the average quality level Q, the expected growth rate is a stationary variable in the transformed model. From the wage-earning household's optimisation problem, we get the following Euler equation:

$$1 = \mathbb{E}_t[\Lambda_{t,t+1}R_{t+1}^b] = \mathbb{E}_t[\Lambda_{t,t+1}R_{t+1}^k] \tag{19}$$

Where the stochastic discount factor Λ is defined as:

$$\Lambda_{t,t+1} = \frac{\tilde{c}_t}{\tilde{c}_{t+1}} \frac{\beta}{(1+g_{t,t+1})}$$

Labour supply can be found from the other first order condition associated with the wage-earning household's optimisation problem:

$$\psi h_t^{-\psi} \tilde{c}_t = \tilde{w}_t \tag{20}$$

Solving the firm's profit function so as to explicitly depend on the production function and mark-up, we get:

$$\pi_t(e) = (\mu - 1) \left[\frac{k_t(e)(R_t^k - 1 + \delta)}{\alpha} \right]^{\alpha} \left[\frac{h_t(e) w_t(e)}{1-\alpha} \right]^{1-\alpha}$$

An increase in the quality level in sector e leads to a productivity gain engendered by that sector's intermediate output of $\lambda^{1-\alpha}$, because of the labour-enhancing nature of quality improvements. This implies that:

$$y_t(q_{i+1}) = \lambda^{1-\alpha} y_t(q_i)$$

Conjecturing that y and k are linear functions of q, we have:

$$y_t q_i = (k_t q_i)^{\alpha} h_t^{1-\alpha} \Rightarrow y_t q_i^{1-\alpha} = k_t^{\alpha} h_t^{1-\alpha} \Rightarrow y_t q_{i+1} = \lambda^{1-\alpha} y_t q_i$$

The same reasoning can be applied to the wage rate, w_t, which means that both capital and wages are linear functions of the sectoral quality level. From the profit expression above, this implies that:

$$\pi_t(k_t q_{i+1}, w_t q_{i+1}) = \pi_t(k_t q_i, w_t q_i) \lambda^{\alpha} \lambda^{1-\alpha} \Rightarrow \pi_t(q_{i+1}) = \lambda \pi_t(q_i)$$

Implying that sectoral profits are linear in the quality level q_e. Firm values are also linear in the quality level because both profits, as has been shown, and research expenditures are linear in q. Therefore, deriving a law of motion for firm value that is stationary in aggregate productivity is straightforward. Equally, the entrepreneur's optimal plans with respect to innovation effort entail that, after taking the integral over all the (symmetric) sectors:

$$1 = \lambda \frac{d\eta(z_t^E)}{dz_t^E} \mathbb{E}_t \left[\frac{\tilde{V}_{t+1}^j(\cdot)}{R_{t+1}^b} \right] \tag{21}$$

While the law of motion for firm value, once again after aggregating over all the sectors, is given by the following expression[9]:

$$\tilde{V}_t(\cdot) = \tilde{\Pi}_t + \left(1 - \eta(z_t^E)\right) \mathbb{E}_t[\Lambda_{t,t+1}\tilde{V}_{t+1}(\cdot)] \tag{22}$$

Assuming that both constraints in the entrepreneur's optimisation problem hold with equality, the budget constraint pins down the equilibrium value of $\tilde{c}_t^E$. Because loans are end of period, their value is tied to the average quality level at $t+1$, and therefore the entrepreneur's stationary budget constraint is :

$$\tilde{c}_t^E + z_t^E = \tilde{L}_{t+1} \tag{23}$$

While the constraint on loan repayments implies that, because end of period loans are tied to the budget constraint at time t, when all the relevant variables are defined in terms of the current quality level q_i and after aggregating across all the sectors:

$$\tilde{L}_{t+1} = \lambda\eta(z_t^E)\mathbb{E}_t\left[\frac{\tilde{V}_{t+1}}{R_{t+1}}\right]$$

On the production side, the equation for capital accumulation is transformed into its stationary equivalent:

$$\tilde{k}_{t+1}(1 + g_{t+1,t}) = (1-\delta)\tilde{k}_t + \tilde{i}_t \tag{24}$$

From the sectoral good producer's optimal choice of inputs, we can easily derive demand for capital and hours worked, which yields the following equations:

$$\alpha\tilde{y}_t = \mu(R_t - 1 + \delta)\tilde{k}_t \tag{25}$$

$$(1-\alpha)\tilde{y}_t = \mu\tilde{w}_t h_t \tag{26}$$

Given the assumptions required to determine the equilibrium outcomes, the production function is reduced to:

$$\tilde{y}_t = u_t\tilde{k}_t^{\alpha}h_t^{1-\alpha} \tag{27}$$

[9]In this context, using either the gross rate of interest or the household's stochastic discount factor is irrelevant because the Euler equation for the wage-earning household dictates that the latter is, in equilibrium, identical to the inverse of the former. The current notation is for the sake of coherence and because, in principle, borrowing rates might diverge in equilibrium from the implied rate from the household's problem in the presence of some form of financial friction or intermediation. I abstract from these considerations here.

And with a fixed price mark-up over marginal cost, the profit function is simply:

$$\tilde{\pi}_t = \frac{\mu - 1}{\mu}\tilde{y}_t \tag{28}$$

The equilibrium growth rate follows directly from 18 and is:

$$g_{t+1,t} = (\lambda - 1)\eta(z_t^E) \tag{29}$$

While the equilibrium resource constraint defined in terms of the transformed stationary variables is:

$$\tilde{y}_t = \tilde{c}_t^E + \tilde{c}_t + \tilde{i}_t + z_t^E \tag{30}$$

Finally, the two laws of motion for the two[10] shocks are as follows:

$$\log(u_t) = \rho_u \log(u_{t-1}) + \varepsilon_{u,t} \quad \text{Productivity shock} \tag{31}$$

$$\log(\eta_t) = \rho_g \log(\eta_{t-1}) + (1 - \rho_g)\eta + \varepsilon_{g,t} \quad \text{Innovation shock} \tag{32}$$

3.2. **Incumbents *versus* Entrants.** The major departure from the standard model discussed earlier occurs by allowing incumbents to engage in research effort of their own. In order to do so, the Arrow replacement effect must be eliminated in order to ensure that, in equilibrium, incumbents will find it profitable to engage in research expenditure. Following Açemoglu and Cao (2010), I assume that incumbents can introduce small improvements to the technology they possess, which have a much higher likelihood of success than the radical innovations brought about by entrants. The details of how innovation happens in this setting are discussed in section (3.2.2).

3.2.1. *Production.* The mechanics of production are identical to the previously discussed model, where final output is a composite of a continuum of intermediate goods, each of these produced by a sectoral monopolist.

[10]In simulating the model, the basic RBC model includes a stochastic growth rate around a deterministic trend. In order to replicate a similar shock in the endogenous growth models, the parameter η is stochastic around the trend defined in table (1).

3.2.2. *Technology and research.* Aggregate quality behaves in exactly the same way as before, in that it is an average of the quality level of each individual sector. In turn, quality in each sector evolves depending on whether the innovation is incremental (the incumbent succeeds in developing a marginal improvement) or drastic (in which case the entrant acquires a radically more productive technology). Incremental improvements in the quality of a given sector then obey the following 'ladder' structure:

$$q_{i+1} = \kappa q_i$$

The interpretation of this is straightforward and follows directly from before: whenever the incumbent is successful, quality jumps by a factor κ onto the next level in the event of a successful discovery. On the other hand, radical improvements are assumed to increase the quality level by a factor of λ, which is large enough to avoid limit pricing[11], when the entrant successfully replaces the incumbent.

Probabilities of success are, as mentioned, different for both. Entrants face the same structure as in the standard model, where the probability of successfully creating a new product, and thereby replacing the incumbent as the monopolist in that sector, is:

$$\Pr(\text{entrant is successful}) = \eta(z_t^E(e))$$

While the incumbent faces the following probability of innovation:

$$\Pr(\text{incumbent is successful}) = \phi(z_t^j(e))$$

Where $\phi(z_t^j(e))$ is a function of incumbent spending. The same assumptions regarding the functional form of $\eta(z_t^E(e))$ still apply and a similar function is used to model the relationship between the probability of success and aggregate research intensities:

$$\phi(z_t^j(e)) = \phi(z_t^j(e))^\gamma, \qquad 0 < \gamma < 1, \quad \phi > 0$$

Expenditure for entrants and incumbent follow the same rules discussed in the previous section: a total expenditure quality-adjusted effort of $z_t^E(e)$ yields the innovation probability above, and the incumbent must also spend a quality-adjusted amount, $z_t^j(e)$, in order to potentially succeed. It is important to note that total expenditure in this

[11]That is, the condition that $\mu < \lambda^{1-\alpha}$ continues to hold in this section.

category is now no longer determined by the entrant's research efforts alone, implying that:

$$Z_t = \int_0^1 \left[z_t^j(e) q_i(e) + z_t^E(e) q_t(e) \right] de$$

As far as the entrant's optimal choice of research effort goes, there is no change relative to the standard model, with equation 14 holding in this case as well. Because incumbents have access to an innovation technology in this version of the model, they must now choose an optimal level of research effort such that they can improve upon the current quality level and remain as the sole producers in that sector. This means that they face the following optimisation problem:

$$V_t(q_i(e)) = \max_{z_t^j(e)} \Bigg\{ \pi_t(q_i(e)) - z_t^j(e) q_i(e) + \phi(z_t^j(e)) \mathbb{E}_t \left[\frac{V_{t+1}(q_{i+1}(e)) - V_{t+1}(q_i(e))}{R_{t+1}^b} \right] + \\ + (1 - \eta(z_t^E(e))) \mathbb{E}_t \left[\frac{V_{t+1}(q_i(e))}{R_{t+1}^b} \right] \Bigg\}$$

The first order condition for the incumbent is then given by:

$$\frac{d\phi(z_t^j(e))}{dz_t^j(e)} \mathbb{E}_t \left[\frac{V_{t+1}(q_{i+1}(e)) - V_{t+1}(q_i(e))}{R_{t+1}^b} \right] = q_i(e)$$

It is worth pointing out that the left hand side corresponds to the marginal benefit of an additional unit of research, while the right hand side corresponds to the unit cost of said research. It starkly differs from the entrant's case because, as per the Arrow effect, the incumbent only gains the amount by which the firm she already owns increases in value. The entrant has nothing and, therefore, if successful, she will gain the entire value of the enterprise. If both had the same probabilities of innovation, then entrants would always be able to outbid the incumbent and the latter would not engage in any research activity.

Given that both entrants and incumbents innovate in this version of the model, the quality level in any given sector may increase on account of either type of innovation. Therefore, assuming a single realisation per period, in every sector it must be that the incumbent is successful in developing an incremental innovation or an entrant develops drastic improvement on the current quality level or neither are successful and the quality level remains the same. Aggregating over all the sectors in the economy and assuming that the probability of success in one sector is independent of the probability of success elsewhere in the economy, the following relationship can be derived:

$$Q_{i+1} \approx \int_0^1 \eta(z_t^E(e))de \cdot \lambda Q_i + \int_0^1 \phi(z_t^j(e))de \cdot \kappa Q_i + \\ + \left(1 - \int_0^1 \eta(z_t^E(e))de - \int_0^1 \phi(z_t^j(e))de\right) Q_i$$

As before, with probability $\eta(z_t^E(e))$, the quality level in sector e increases by λ. Ties are avoided by requiring that the measure of entrepreneurs be identical to the measure of sectors, while individual probabilities of success are assumed to be small enough that $\eta(z_t^E(e)) \cdot \phi(z_t^j(e)) \approx 0$. This means the growth rate for aggregate productivity can be approximated by:

(33)
$$(1 + g_{t,t+1}) = \frac{Q_{i+1} - Q_i}{Q_i} \approx (\lambda - 1)\eta(z_t^E) + (\kappa - 1)(z_t^j)$$

3.2.3. *Equilibrium.* The only change from the model outlined in the previous section comes from allowing incumbents to engage in innovative activity of their own. Production, capital accumulation and household decisions are unaffected, as are the optimal research efforts of entrepreneurs, but firm values now depend on whether incumbents succeed in developing their own innovations, while the growth rate for the endogenous component of aggregate productivity and aggregate research expenditure are also different.

Starting with latter, equation (29) is replaced by (33), which now reflects incumbents' average contribution to productivity growth. Additionally, total research expenditure is now defined as the sum of research efforts by both entrepreneurs and incumbents. This means:

(34)
$$z_t = z_t^E + z_t^j$$

The optimal amount of research effort by incumbents is given following equilibrium relationship:

(35)
$$1 = \frac{d\phi(z_t^j)}{dz_t^j}\mathbb{E}_t\left[\frac{\tilde{V}_{t+1}\kappa - \tilde{V}_{t+1}}{R_{t+1}^b}\right] = (\kappa - 1)\frac{d\phi(z_t^j)}{dz_t^j}\mathbb{E}_t\left[\frac{\tilde{V}_{t+1}}{R_{t+1}^b}\right]$$

For the average incumbent, the optimal decision to invest in research and development depends on the additional gain coming from successful innovations, which in this case is embodied in the quality jump parameter κ. Finally, the recursive representation for the average value of a firm in production is given by:

$$\tilde{V}_t = \tilde{\pi}_t - z_t^j + \phi(z_t^j)(\kappa - 1)\mathbb{E}_t\left[\frac{\tilde{V}_{t+1}}{R_{t+1}^b}\right] + (1 - \eta(z_t^E))\mathbb{E}_t\left[\frac{\tilde{V}_{t+1}}{R_{t+1}^b}\right] \tag{36}$$

3.3. **Innovators *versus* imitators.**

3.3.1. *Resource constraints.* These take on the same form discussed in the first and second variants of the model, with the only difference between the first and the last two models coming from incumbeents engaging in research expenditure.

3.3.2. *Production.* Given the significant differences between this model and the ones discussed in the previous sections, it is worthwhile to explore the differences in how production takes place. There is a single final good which is the result of aggregating all sectoral goods according to the following production function:

$$Y_t = \left[\int_0^1 u_t \omega_t(e) y_t(e)^{\frac{\mu-1}{\mu}} dm\right]^{\frac{\mu}{\mu-1}}$$

$$y_t(e) = \int_0^{n_t(e)} y_t^j(e) dj$$

Where $\omega_t(e)$ is as defined in (7). Although this functional form shares some similarities with that of the baseline model, it must be noted that each sectoral output is in turn produced by $n_t(m)$ identical firms who compete à lá Cournot. This means that instead of there being a single monopolist in each sector, there is now a finite number of firms that compete in the production and sale of the sectoral good e. The production function for the j-th of these n_e firms is:

$$y_t^j(e) = [k_t^j(e)]^\alpha [h_t^j(e)]^{1-\alpha}$$

The assumption of competition in quantity, à lá Cournot, yields a price mark-up that is a function of the price elasticity of the sectoral output as well as the number of firms in each market. This closely follows the approaches developed in Galí and Zilibotti (1995),[12],

[12]A more detailed exposition of how these price mark-ups can be generated, including Bertrand and Stackelberg versions can be found in Etro (2012).

which means the price mark-up is given by:

$$p_t(e) = \frac{\mu n_t(e)}{\mu n_t(e) - 1} mc_t^j(e)$$

Where $mc_t^j(e)$ is the composite marginal cost faced by producer j. The implication is clear: as the number of firms in sector e increases, it tends to the competitive outcome of a zero mark-up over marginal cost. Under the assumption that sectoral output is evenly divided between all market participants, we can write each individual firm's profit function as:

$$\pi_t^j(e) = \frac{p_t(e) y_t^j(e)}{\mu n_t(e)}$$

The optimal choice of inputs and the marginal cost of production for a single intermediate good producer are the same as in the previous model because the production technology and rental prices of capital and labour services are still the same.

3.3.3. *Technology and research.* The ladder structure and expression for the average quality level are retained from the models discussed in the preceding sections. The main difference lies in the fact that rather than having entrants engage in "creative destruction" and incumbents in piecemeal innovation, we assume that instead of doing so, incumbents can attempt to imitate the new inventions developed by entrants. In other words, an entrant will develop a radical new technology which in the previous model would displace the incumbent and create a new monopoly position, while incumbents would generate piecemeal innovations in order to strengthen their own existing monopoly.

Instead, I assume that when a radical innovation takes place, incumbents can engage in 'imitation effort' preemptively and, if successful at copying this new technology, remain in the market as a competitor to the innovator. The rationale for this mechanism is straightforward: while retaining the creative destruction feature of standard Schumpeterian models, I introduce a more flexible market structure that allows incumbents to stay in production throughout relatively long waves of successive innovations. In short, both departures from the standard Schumpeterian model introduce a higher average lifetime for any given firm: the Açemoglu and Cao (2010), version by allowing monopolists a change to retain that status for longer and the present model of innovation and imitation by allowing incumbents to compete on an equal footing with new entrants.

One can find examples such behaviour relatively easily: a new firm introduces a new product into the market and soon enough existing industry stalwarts come forward with their own versions of the product. More importantly, however, if one defines the value of a firm as the discounted present value of the stream of future profits from a given product line, then one can conceive of each sector as an independent product line in which an entrant may well be a well established firm in other product lines. Under that interpretation, potential examples for the industry dynamics described here are potentially limitless.

Formally then, we can begin by characterising the behaviour of key aggregates. The quality level evolves according to:

$$q_{i+1} = \lambda q_t$$

Where it is important to note that the research expenditure of incumbents does not advance the overall quality level. Rather, it only allows the incumbent, if successful, to stay in the market should a potential entrant successfully develop a new product. Therefore, it does not contribute to economic growth but instead contributes towards the allocative efficiency of the economy: by eroding market power, it ensures that more output is produced and at a lower price. This establishes an economy wide trade off between economic growth and the imitation efforts of incumbents because the erosion of market power generated by the latter is likely to reduce genuine innovation through less expenditure on the part of entrepreneurs.
The modelling for this mechanism borrows liberally from the examples explored above. Total expenditure in research & development by a potential entrant takes the same form as before, $z_t^E(e)q_i$, and is justified by similar arguments: a higher degree of technological sophistication requires more research effort. This leads to a familiar expression for the probability of success:

$$\Pr(\text{innovation is successful}) = \eta(z_t^E(e)) = \eta(z_t^E(e))^\gamma, \qquad 0 < \gamma < 1, \quad \eta > 0$$

Again, $\eta(\cdot)$ is strictly increasing in $z_t^E(e)$, which is the total amount of research & expenditure undergone by entrepreneurs in that sector. In order to absorb some of the notation and structure of the second model, the imitation process is formalised in the likeness of the piecemeal innovations by incumbents discussed in said model. Hence, an incumbent must spend an amount of $z_t^j(e)q_i$, which gives her a probability of imitating of:

$$\Pr(\text{imitation is successful}) = \phi(z_t^j(e)) = \phi(z_t^j(e))^\gamma, \qquad 0 < \gamma < 1, \quad \phi > 0$$

Total research and development is still defined as the sum between research expenditure by both entrants and incumbents and is given by:

$$Z_t = \int_0^1 \left[z_t(e)q_i(e) + \hat{z}_t^E(e)q_i(e)\right] de$$
$$\text{where} \quad z_t(e) = \int_0^{n_t(e)} z_t^j(e)dj$$

The entrepreneur's optimal choice of innovation effort is governed by equation (14), with the value of any potential new firm created following a successful innovation depending on the number of firms in activity during the following period. This means that any potential entrant must take into account the fact that when a new innovation arrives, current incumbents will attempt to copy that technology and, if successful, will also compete with the innovator in the following period.

In turn, the incumbent faces a much more complex decision of her own. Under the assumption that should she choose to engage in research in radical innovations, she would face the same probability of innovation and cost structure of an entrepreneur, it is clear that the Arrow replacement effect explicitly outlined in equation (17) would imply zero research expenditure from the incumbent[13]. This leaves only the option of attempting to copy any successful innovation, which leads her to the following recursive formulation for the value of an incumbent firm:

$$V_t^j(q_i(e)) = \pi_t^j(q_i(e)) - z_t^j(e)q_i(e) + \eta(z_t^E(e))\phi(z_t^j(e))\mathbb{E}_t\left[\Lambda_{t,t+1}V_{t+1}^j(q_{i+1})\right] +$$
$$+(1-\eta(z_t^E(e)))\mathbb{E}_t\left[\Lambda_{t,t+1}V_{t+1}^j(q_i)\right]$$

This warrants further discussion. The first thing to notice is that an incumbent can only be successful in imitating should there be a successful innovation. In other words, she

[13]In other words, we're back in a world in which incumbents cannot innovate because they possess the same invention generating technology, but have the ability to, through their research expenditure, copy newly developed technologies.

can only imitate a new technology if it exists. If not, she remains in the market and there is no change to the firm's expected value[14]. If the entrepreneur is successful, however, the incumbent either imitates the technology and therefore retains incumbent status or she fails to do so and loses the entire value of the firm. This is capture in the recursive equation for firm value: if the quality level increases with probability $\eta(\cdot)$, then with probability $\phi(\cdot)$ an incumbent will retain that status and benefit from the productivity improvement. With probability $1-\phi(\cdot)$, the incumbent fails in developing an alternative and is priced out of the market with its current technology. An incumbent's optimal decision regarding the level of research and development to undergo is then straightforward to derive:

$$\eta(z_t^E(e))\frac{d\phi(z_t^j(e))}{dz_t^j(e)}\mathbb{E}_t\left[\Lambda_{t,t+1}V_{t+1}^j(q_{i+1}(e))\right]=q_i(e)$$

The final element to be introduced is how the number of firms evolves throughout time. This expected number of firms is given by the probability of an innovation occurring within the time period, which if realised yields a number of firms at time $t+1$ of one innovator plus all of those incumbents that succeeded in copying the technology; and the probability of no innovation arriving, which yields the same number of firms as before. Formally:

$$\mathbb{E}_t[n_{t+1}(e)]=\eta(z_t^E(e))\left(1+\int_0^{n_t(e)}\phi(z_t^j(e))dj\right)+\left(1-\eta(z_t^E(e))\right)n_t(e)$$

Because all other elements in the economy are identical to the previously discussed frameworks, aggregation and equilibrium follow straightforwardly.

3.3.4. *Aggregation and equilibrium.* Again, changes to the structure of the model generate slightly different equilibrium outcomes. I outline here the main departures from the previous section. Economic growth now depends exclusively on the research effort of entrepreneurs, which means that the expression for g is again given by equation (29).

Aggregation of all the variables must now take into account the fact that each sector is populated with $n_t(e)$ different firms. Hence, $C_t=\int_0^1 c_t(\iota)d\iota$, $s=\int_0^1 s(\iota)d\iota=1$, $h_t=\int_0^1 h_t(\iota)d\iota=\int_0^1\int_0^{n_t(e)}h_t^j(e)djde$, $\Pi_t=\int_0^1\int_0^{n_t(e)}\pi_t^j(e)djde$, $K_t=\int_0^1 k_t(\iota)d\iota=\int_0^1\int_0^{n_t(e)}k_t^j(e)djde$, $L_t=\int_0^1 B_t(\iota)d\iota=\int_0^1 L_t(e)de$, $\eta(z_t^E)=\int_0^1\eta(z_t^E)de$, $V_t=\int_0^1\int_0^{n_t(e)}V_t^j(e)djde$,

[14]Evidently, economic conditions may be different in the following period, but this should not be the case in terms of expected values.

$C_t^E = \int_0^1 c_t^E(e)de$, $z_t = \int_0^1 z_t^E(e)de + \int_0^1 \int_0^{n_t(e)} z_t^j(e)djde$, $n_t = \int_0^1 n_t(e)$.

Production decisions at the individual firm are identical, as are the decisions of both types of households, so all the relevant equations from section (3.1.3) apply here. As identified in the preceding section, the major changes pertain to how profits and firm values are distributed and determined in equilibrium. Aggregate (which are paid to the wage earning households) and firm profits are now given by (after aggregating and transforming them into their stationary equivalent)[15]:

$$\tilde{\Pi}_t = n_t \tilde{\pi}_t^j \tag{37}$$

$$\tilde{\pi}_t^j = \frac{\tilde{y}_t}{\mu(n_t)^2} \tag{38}$$

As is clear, aggregate profits will be a simple sum over the n_t average number of firms, and firm profits will decrease more than proportionally with the number of firms. This is because sectoral output is not only evenly spread amongst the firms in each sector but also because the mark-up over marginal cost will decrease as the number of competitors increases. The aggregate law of motion for this average number of firms is then given by:

$$n_{t+1} = \eta(z_t^E)\left(1 + \int_0^{n_t} \phi(z_t^j)dj\right) + \left(1 - \eta(z_t^E)\right) n_t \tag{39}$$

Finally, from the optimality conditions for incumbents and entrepreneurs, we get the following equilibrium relationship between optimal research by the two types of agents:

$$\eta(z_t^E)\frac{d\phi(z_t^j)}{dz_t^j}(\kappa - 1)\mathbb{E}_t[\Lambda_{t,t+1}\tilde{V}_{t+1}^j] = \frac{d\eta(z_t^E)}{dz_t^E}\lambda\mathbb{E}_t\left[\frac{\tilde{V}_{t+1}^j}{R_{t+1}^b}\right] \tag{40}$$

The latter condition can be interpreted as a free entry condition that pins down the optimal amount of imitation effort in which incumbents engage in, given how much entrepreneurs invest in turn.

4. Calibration

4.1. **Model setup.** Using US data from the Bureau of Economic Analysis' National Income and Product Accounts tables, all versions of the endogenous productivity model are calibrated to generate a steady-state growth rate of real output of 0.7%, a ratio of

[15]Note that because the probabilities of innovation and imitation are themselves stationary, it follows that the number of firms is going to be stable in equilibrium.

private research expenditure to output of 1.32% and a price mark-up of 1.3, as reported by Jaimovich and Floetotto (2008). Table (1) contains all the parameter values for four different models: a benchmark RBC model, an RBC model with endogenously determined productivity growth, an RBC model in which incumbents can contribute to innovation efforts and a model with endogenously determined mark-ups and productivity growth.

TABLE 1. Structural parameters

Parameters			**Values**		**Description**
α	0.33	0.33	0.33	0.33	Capital share
β	0.995	0.995	0.995	0.995	Discount factor patient households
γ	–	0.071	0.093	0.043	Research elasticity
δ	0.025	0.025	0.025	0.025	Depreciation rate
η	–	0.020	0.015	0.019	Scaling parameter, innovation
ϕ	–	–	0.034	0.640	Scaling parameter, imitation
μ	1.3	1.3	1.3	2.1(6)	Demand elasticity
ψ	1.3(3)	1.3(3)	1.3(3)	1.3(3)	Inverse Frisch elasticity labour
φ	0.179	0.136	0.140	0.142	Weight labour in utility
λ	–	1.482	1.482	1.482	Quality jump
κ	–	–	1.121	–	Quality jump
ρ_u	0.95	0.95	0.95	0.95	Shock persistence
$\sigma_{u,t}$	0.007	0.007	0.007	0.007	Size of shock
ρ_g	0.95	0.95	0.95	0.95	Shock persistence
σ_g	0.01	0.01	0.01	0.01	Size of shock
Model	Exo RBC	Endo RBC	Incumb RBC	EndoMkUp RBC	

Some of the parameters are lifted directly from the DGSE literature, with α set to 0.33, the discount factor for both types of household, β, set to 0.995, and the quarterly rate of depreciation, δ, is set at 0.025. All of these are taken from Brzoza-Brzezina et al. (2013). The inverse of the Frisch elasticity for labour supply, ψ, is set at 4/3 following the recommended estimates for aggregate hours in Chetty et al. (2011). The remaining parameters are allowed to vary according to the version of the model to match the aforementioned first moments of US aggregate data. The size of the quality jump of entrepreneur-driven innovation is set at 1.482 to ensure that limit pricing does not occur.

Table (2) summarises a few significant steady state results, showing how the various models carry different implications for firm valuations. In particular, firm values are higher in the model in which incumbents are allowed to innovate relative to the standard endogenous growth model because they price in potential future gains accruing from successful innovations. Likewise, the model with endogenous market entry, profits for individual firms are smaller, but aggregate profits are identical to those in preceding models, while individual firm valuations reflect the higher probability of staying in the market because of incumbents' ability to imitate technological breakthroughs.

TABLE 2. Steady state

Variables	**Data**	**Exo RBC**	**Endo RBC**	**Incumb RBC**	**EndoMkUp RBC**
Output	–	1.032	1.032	1.034	1.032
Profits	–	0.238	0.238	0.239	0.119
Firm value	–	–	9.063	12.415	7.395
Research share in output	0.0132	–	0.0132	0.0132	0.0132
Steady state growth rate	0.007	0.007	0.007	0.007	0.007
Endogenous mark-up	–	‡	‡	1.3	1.3
Entry rate	–	–	0.015	0.01	0.015
Investment share	0.18	0.219	0.219	0.219	0.219

‡ Equal to the elasticity of substitution, μ.

4.2. **Results.** All the models simulated in this section are compared to data for the period between the first quarter 1970 and the last quarter of 2008, with the cutoff point chosen so as to remove the effects of the Great Recession, given that the recovery from that recessionary episode is still ongoing. Table (3) presents key moments in the for the four aggregate expenditure variables in the model: gross domestic product, consumption, investment and research spending. These are calculated using four different specifications for two different filters: the Hodrick-Prescott filter and the Baxter-King band pass filter. The H-P filter is standard in the business cycle literature and is often associated with the higher-frequency fluctuations that form the bulk of cyclical analysis, while the band pass filter is used to extract information for two different specifications of the 'medium-term fluctuations': the one proposed by Phillips and Wrase (2006) for periodicities between 20 and 80 quarters, and two specifications due to Comin and Gertler (2006) for periodicities between 32 and 200 quarters as well as periodicities between 2 and 200 quarters, encompassing the entire length of the dataset[16].

[16]The dataset contains only 156 period observations, but using bk(2,156) and bk(32,156) or the amplitude mentioned in the text makes no difference as the length of the dataset is contained within that amplitude.

Table 3. Moments from US data

US data, 1970q1 to 2008q4									
	HP filter, $\lambda = 1600$					Baxter-King, $(20, 80)$			
	Y	C	Z	I		Y	C	Z	I
σ	1.544	1.296	2.267	6.504	–	1.084	0.943	1.784	4.341
σ/σ_Y	1	0.839	1.469	4.214	–	1	0.869	1.645	4.004
ρ_{X_t,Y_t}	1	0.873	0.452	0.924	–	1	0.918	0.410	0.935
	Baxter-King, $(32, 200)$					Baxter-King, $(2, 200)$			
	Y	C	Z	I		Y	C	Z	I
σ	0.393	0.343	0.651	1.564	–	1.796	1.494	2.710	7.710
σ/σ_Y	1	0.873	1.657	3.979	–	1	0.832	1.509	4.292
ρ_{X_t,Y_t}	1	0.917	0.406	0.932	–	1	0.878	0.432	0.929

Including both specifications for 'medium-term' fluctuations is justified by briefly appealing to the results in table (3): the approach used in Phillips and Wrase (2006) overlaps the standard 6 to 32 period business cycle amplitude, while the one proposed in Comin and Gertler (2006) removes most of the higher frequency variability. Using both ensures that the results are more general and not tied to a specific definition of the medium-term business cycle.

Because the models include either deterministic, exogenously determined trends (in the case of the RBC model) or endogenously determined paths for labour productivity, I follow Phillips and Wrase (2006) in reconstructing the non-stationary series by multiplying the simulated values for each variable with the average labour productivity level[17]. This yields non-stationary time series for all variables of interest, to which the same filters used in the data are then applied.

The standard deviation of the labour productivity shock is taken from BLS data, rather than chosen to match the volatility of output in the data, while the standard deviation of the growth shock is arbitrarily set at 1%. Therefore, rather than focusing on how

[17]Given an initial quality level Q_0, here set at 1, Q_1 is defined as: $Q_1 = Q_0(1 + g_1)$, where g is the deterministic (in the RBC model) or endogenously determined growth rate (in the remaining three models).

TABLE 4. Moments simulated benchmark RBC model, two shocks

RBC model with exogenous growth rate, simulated for 156 periods									
	HP filter, $\lambda = 1600$					Baxter-King, $(20, 80)$			
	Y	C	Z	I		Y	C	Z	I
σ	1.195	0.515	.	3.800	–	0.659	0.337	.	1.995
σ/σ_Y	1	0.431	.	3.181	–	1	0.511	.	3.025
ρ_{X_t,Y_t}	1	0.935	.	0.986	–	1	0.887	.	0.964
	Baxter-King, $(32, 200)$					Baxter-King, $(2, 200)$			
	Y	C	Z	I		Y	C	Z	I
σ	0.243	0.125	.	0.732	–	1.269	0.561	.	4.015
σ/σ_Y	1	0.515	.	3.017	–	1	0.442	.	3.163
ρ_{X_t,Y_t}	1	0.886	.	0.963	–	1	0.923	.	0.983

models perform in terms of matching moments in the data, I rather focus on inter-model comparisons. Motivating this choice is the fact that because the underlying structure in all of the models is that of a basic real business cycle model without any frictions, they are all expected to do reasonably poorly in terms of matching those moments. If, however, explicitly modelling endogenous productivity growth matters in terms of generating a time path for key macroeconomic variables, the performance of models with endogenous growth relative to that of the exogenous growth RBC model should be noticeably better. All the same, table (4) highlights the benchmark RBC model's lower volatilities of output, consumption and investment when allowing for shocks to both labour productivity and the exogenous growth rate.

Including a simple growth generating process through research expenditure does very little to improve on the size of the volatilities of output and consumption, while the volatility of investment is even lower. There is some improvement at lower frequencies but, overall, the model does generate any significant additional volatility by endogenising the growth generating mechanism.

The same pattern emerges when allowing both entrants and incumbents to innovate, with all major volatilities virtually identical to the model where only entrants are allowed

Table 5. Moments simulated endogenous growth RBC model, two shocks

RBC model with endogenous growth, simulated for 156 periods									
	HP filter, $\lambda = 1600$					Baxter-King, $(20, 80)$			
	Y	C	Z	I		Y	C	Z	I
σ	1.203	0.566	1.202	3.706	–	0.657	0.350	0.607	1.988
σ/σ_Y	1	0.470	0.999	3.080	–	1	0.533	0.924	3.027
ρ_{X_t,Y_t}	1	0.923	0.616	0.977	–	1	0.868	0.480	0.952
	Baxter-King, $(32, 200)$					Baxter-King, $(2, 200)$			
	Y	C	Z	I		Y	C	Z	I
σ	0.241	0.129	0.225	0.729	–	1.271	0.601	1.160	3.966
σ/σ_Y	1	0.536	0.932	3.018	–	1	0.473	0.913	3.121
ρ_{X_t,Y_t}	1	0.868	0.481	0.952	–	1	0.903	0.544	0.972

Table 6. Moments simulated model incumbent innovation, two shocks

RBC model with endogenous growth, simulated for 156 periods									
	HP filter, $\lambda = 1600$					Baxter-King, $(20, 80)$			
	Y	C	Z	I		Y	C	Z	I
σ	1.204	0.558	1.044	3.731	–	0.657	0.349	0.539	1.991
σ/σ_Y	1	0.463	0.867	3.098	–	1	0.531	0.820	3.031
ρ_{X_t,Y_t}	1	0.923	0.668	0.978	–	1	0.868	0.554	0.953
	Baxter-King, $(32, 200)$					Baxter-King, $(2, 200)$			
	Y	C	Z	I		Y	C	Z	I
σ	0.242	0.129	0.200	0.731	–	1.271	0.595	1.018	3.981
σ/σ_Y	1	0.534	0.826	3.021	–	1	0.468	.801	3.132
ρ_{X_t,Y_t}	1	0.868	0.555	0.952	–	1	0.904	0.607	0.973

to innovate with the exception of that of research expenditure, which is lower than in the previous variant at all frequencies.

Finally, the model with endogenous entry generates identical results for output, consumption and investment while improving marginally on the preceding two models in terms of the volatility of research expenditure. The overall fit of all the models is summarised in table (8), which reports the root mean squared deviation for all the moments

TABLE 7. Moments simulated model endogenous entry, two shocks

RBC model with endogenous growth and entry, simulated for 156 periods									
	HP filter, $\lambda = 1600$					Baxter-King, $(20, 80)$			
	Y	C	Z	I		Y	C	Z	I
σ	1.201	0.554	1.307	3.737	–	0.656	0.346	0.665	2.001
σ/σ_Y	1	0.461	1.088	3.111	–	1	0.528	1.013	3.048
ρ_{X_t,Y_t}	1	0.921	0.556	0.977	–	1	0.866	0.401	0.953
	Baxter-King, $(32, 200)$					Baxter-King, $(2, 200)$			
	Y	C	Z	I		Y	C	Z	I
σ	0.242	0.128	0.247	0.734	–	1.269	0.591	1.264	3.995
σ/σ_Y	1	0.531	1.021	3.038	–	1	0.465	0.996	3.148
ρ_{X_t,Y_t}	1	0.866	0.402	0.952	–	1	0.901	0.471	0.973

reported in tables (3) through (7). In comparison with the benchmark RBC model with exogenous growth, all the models with endogenous productivity growth perform worse at the standard frequencies used in the business cycle literature, suggesting that not only does incorporating endogenous productivity growth not lead to an improvement over a more parsimonious version in which growth is taken as an exogenous quantity, it performs slightly worse. Over the medium-term frequencies, however, that performance differential is reversed, with all endogenous growth models doing slightly better and the model with both endogenous productivity growth and endogenous market structure outperforming all other three variants.

This improvement in performance is not unexpected, as both Comin and Gertler (2006), and Phillips and Wrase (2006), report on the suitability of models that incorporate endogenous productivity growth in replicating moments at lower frequencies. Surprisingly, however, the gains in performance relative to a simple exogenous growth RBC model are small, and over the entire frequency of the data the loss of performance in the standard business cycle frequencies outweighs those improvements. Calculating the RMSD with research expenditure improves on the performance of all the models with endogenous growth, but because the standard RBC model does not allow for research expenditure, comparisons with its performance are invalid. In this case, the gain in performance of

Table 8. Model fit

Model fit using moments from tables (3) to (7), RMSD				
	Exo RBC	Endo RBC	Incumb RBC	EndoMkUp RBC
Without research expenditure				
HP, $\lambda = 1600$	1.152*	1.190	1.180	1.177
BK, $[20, 80]$	1.010	1.010	1.009	1.003*
BK, $[32, 200]$	0.510	0.508	0.507	0.502*
BK, $[2, 200]$	1.523*	1.540	1.534	1.528
With research expenditure				
HP, $\lambda = 1600$	–	1.062	1.079	1.038*
BK, $[20, 80]$	–	0.951	0.950	0.933*
BK, $[32, 200]$	–	0.502	0.521	0.483*
BK, $[2, 200]$	–	1.392	1.409	1.367*

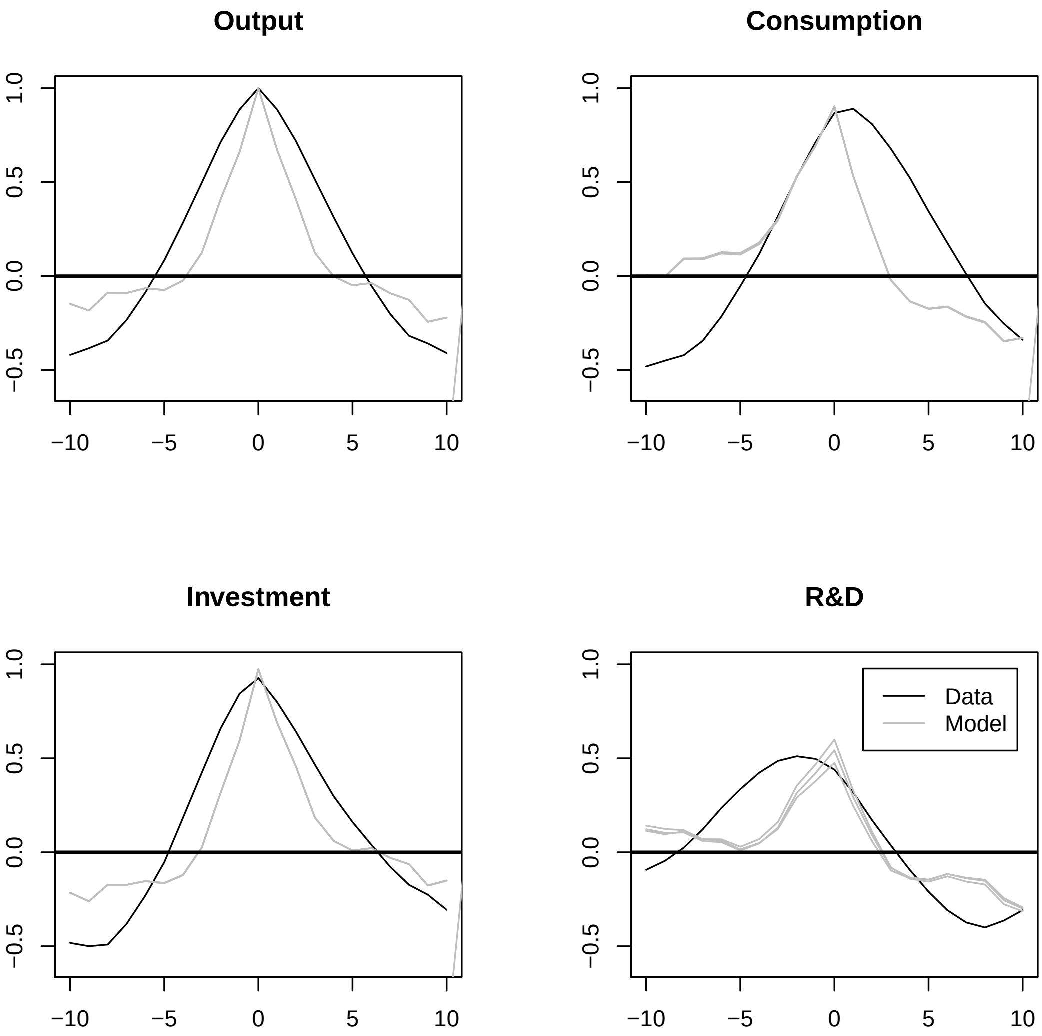
the model with endogenous growth and endogenous mark-ups comes from doing slightly better at matching the volatility of research expenditure and its correlation with output at all frequencies.

Analysing figure (2), it is clear that all the models behave identically when it comes to replicating the cyclical comovement of key variables. Output displays insufficient autocorrelation, while consumption leads the cycle despite all models predicting that it ought to lag output[18]. The contemporaneous correlation between investment and output is captured equally well across all variants of the model, but again the pattern of that correlation with past and future values of output is not well captured by any of them. Finally, all the models featuring endogenous productivity growth fare better in capturing the correlation between research spending and output, but at higher frequencies predict the correlation between research spending and past values of output is weaker than what the data indicates. Finally, at 'medium term' frequencies[19], all the models do substantially better in matching the autocorrelation of output and investment, though continue to predict

[18]See Wen (2001) for a more detailed discussion.

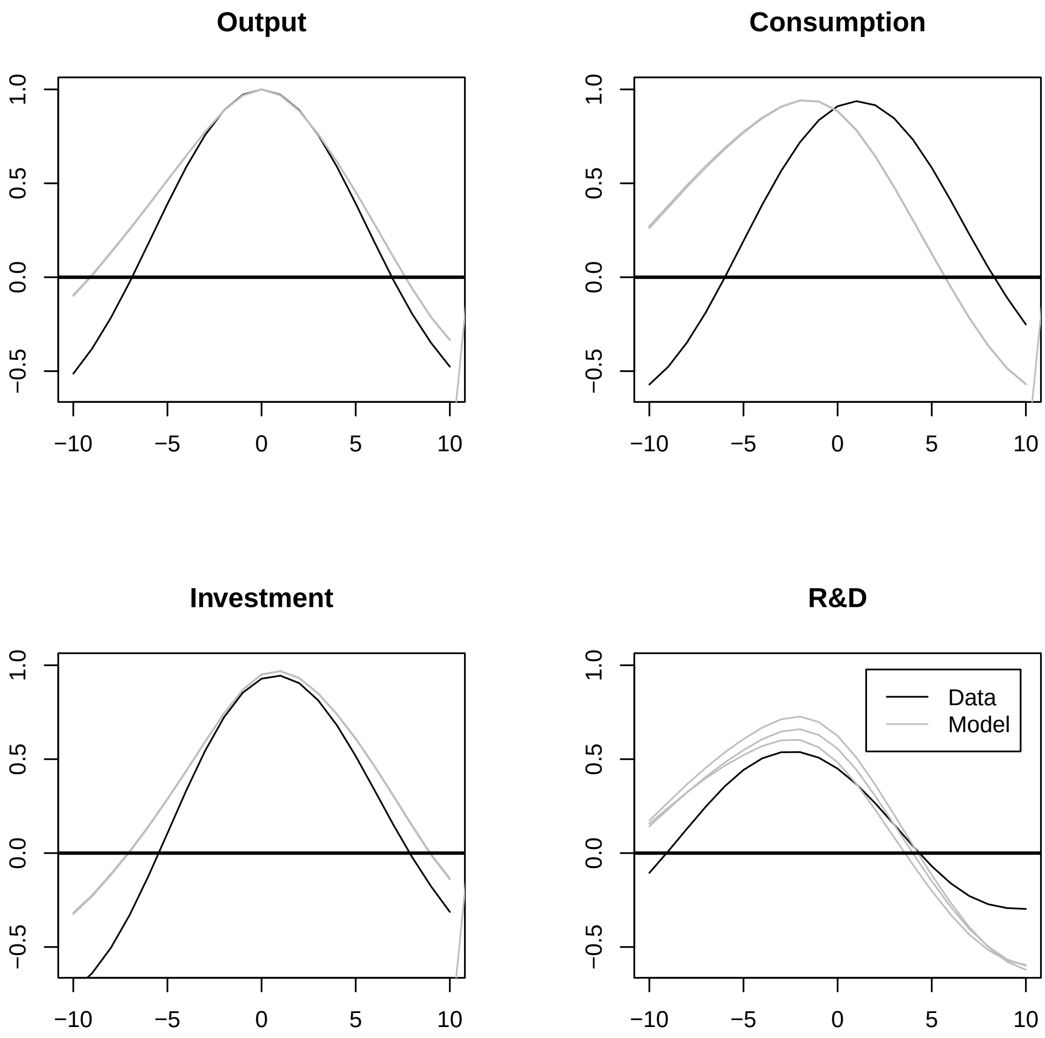
[19]I choose to display the series filtered using 32 and 200 periods as end points only to facilitate the presentation of results because the behaviour of the comovements using the alternative 20 and 80 are identical.

FIGURE 2. Correlations with output, HP filter $\lambda = 1600$



lagging, rather than leading, consumption. At this frequency, despite the gains in performance highlighted in table (8) for the models incorporating endogenously determined growth, those gains are reasonably small and all variants still significantly underperform for correlations at longer distances from period t and in capturing the behaviour of consumption. All models with endogenous growth fare substantially better in capturing the behaviour of R&D expenditure at lower frequencies (bottom right graph in picture (3)),

FIGURE 3. Correlations with output, BK filter [32,200]



with the model with endogenous market structure outperforming the models with only entrant innovation or entrant and incumbent innovation.

5. DISCUSSION

An inescapable conclusion from this exercise is that the simple addition of an endogenous growth generating mechanism fails to substantially improve significantly on a basic RBC model in terms of capturing main features of the business cycle, which in turn implies that the underlying propagation mechanism of the neoclassical model is mostly unaffected by explicitly linking future output growth with research expenditures. To an extent, this is understandable: the endogenous productivity generating mechanism simply subtracts a certain quantity from output today in order to generate future growth, while the standard, exogenous growth framework simply assumes a growth rate that is a stochastic variable. This is not very dissimilar from what the endogenous growth mechanism generates, and therefore models with and without this feature turn out to behave very similarly.

However, the existence of a feedback mechanism between output, research spending and growth should, at least in principle, have an effect on the behaviour of future output: large fluctuations in output should generate fluctuations in research expenditure[20] that impact future growth prospects. In the model, that feedback mechanism is dampened by the parameter governing the elasticity of the probability of an innovation with respect to research spending taking on a very low value. Because of the parsimonious nature of the growth generating mechanism outlined here, this parameter is calibrated to match observed quarterly growth rates of output and therefore different calibrations with higher values may strengthen this effect. Running the model for different values without recalibrating it to match observed first moments does not lead to significantly higher output volatility and somewhat improves the autocorrelation of output for periods further away, but does little to improve the performance of the model at shorter frequencies.

The labour productivity shock accounts for virtually all of the variation in output, consumption and investment both in the exogenous growth RBC model and in all the variants with endogenous growth, suggesting that even in the presence of a shock to growth prospects in the former, or a shock to the scaling parameter in the probability of innovation in the latter case, the feedback mechanism between output and growth is very weak and does not significantly affect the internal propagation mechanism of the RBC model. In addition, the growth/probability shock only affects the growth rate and levels of research expenditure for small sized shocks[21], implying that in order for shocks to the growth rate in either version to have any effect on other variables, these have to be quite

[20]Both data and the models suggest that research expenditure lags the cycle.

[21]The value used in the model simulations here is 1%.

substantial in size. In short, the labour productivity shock is the primary driving force for the model's internal propagation mechanism and explicitly allowing for a feedback between output and output growth does not substantially affect the performance of the model.

6. Conclusion

The assumption that cyclical fluctuations and growth are two separable and reasonably independent phenomena has long been implied by the thin overlap between the dominant modelling strategies in macroeconomics that attempt to account for either, but the twin questions of whether growth is important for cycles and whether cycles affect growth has gained importance. Previous work in the field suggests that including endogenous growth elements in the form of learning by doing or human capital accumulation in standard business cycle models had significant implications in terms of these models' ability to generate accurate statistical representations of key macroeconomic variables.

The lack of a dominant and widely accepted primary growth generating mechanism means, however, that these results may not be general if the contribution of learning-by-doing or human capital to aggregate productivity growth is dominated by the process of replacing old technologies and firms with new ones through creative destruction. Furthermore, although labour may be a counter-cyclical input to the process of productivity growth, overall research spending display a pro-cyclical bias that models relying exclusively on these mechanisms must abstract from.

I propose three simple variants of a quality ladder growth model in which research spending is the only input into the innovation generating process and embed these in a standard real business cycle growth model. Comparing the performance across all modelling variations, it is clear that in this simple form, the internal propagation mechanism of the RBC model is largely unaffected by the growth components and that the direct impact of exogenous productivity shocks on output and other key expenditure variables almost completely outweighs the growth feedback mechanism. At the higher frequencies associated with the business cycle literature, the RBC model outperforms all the alternatives in terms of matching key moments in the data. Extending the analysis to lower frequencies, I find that the endogenous growth models slightly outperform the model with a deterministic growth rate, but that this performance gain is small relative to the loss generated at higher frequencies. However, all endogenous growth models do comparatively well at capturing the comovement of research expenditure across the cycle, which the RBC model is by construction incapable of capturing.

These results suggest that abstracting from this type of growth generating mechanism is likely to have very limited impact on the ability of simple business cycle models to match observed empirical facts, but the possibility remains that in models with more sophisticated internal propagation mechanisms, nominal frictions or financial frictions, the interaction between these and the growth generating components may lead to very different conclusions. Future research should attempt to explicitly incorporate these elements into richer environments so as to more conclusively settle the issue of whether the feedback mechanism between growth and cycles is as weak as is suggested here.

References

Daron Açemoglu. Introduction to modern economic growth. Levine's Bibliography 122247000000001721, UCLA Department of Economics, November 2007. URL http://ideas.repec.org/p/cla/levrem/122247000000001721.html.

Daron Açemoglu and Dan Vu Cao. Innovation by entrants and incumbents. NBER Working Papers 16411, National Bureau of Economic Research, Inc, September 2010. URL http://ideas.repec.org/p/nbr/nberwo/16411.html.

Galo Nuño Barrau. Optimal research and development expenditure: a general equilibrium approach. Banco de España Working Papers 1009, Banco de España, March 2010. URL http://ideas.repec.org/p/bde/wpaper/1009.html.

Michał Brzoza-Brzezina, Marcin Kolasa, and Krzysztof Makarski. The anatomy of standard dsge models with financial frictions. *Journal of Economic Dynamics and Control*, 37(1):32–51, 2013. URL http://ideas.repec.org/a/eee/dyncon/v37y2013i1p32-51.html.

Raj Chetty, Adam Guren, Day Manoli, and Andrea Weber. Are micro and macro labor supply elasticities consistent? a review of evidence on the intensive and extensive margins. *American Economic Review*, 101(3):471–75, September 2011. doi: 10.1257/aer.101.3.471. URL http://www.aeaweb.org/articles.php?doi=10.1257/aer.101.3.471.

Diego Comin and Mark Gertler. Medium-term business cycles. *American Economic Review*, 96(3):523–551, June 2006. URL http://ideas.repec.org/a/aea/aecrev/v96y2006i3p523-551.html.

Diego Comin, Mark Gertler, and Ana Maria Santacréu. Technology innovation and diffusion as sources of output and asset price fluctuations. NBER Working Papers 15029, National Bureau of Economic Research, Inc, June 2009. URL http://ideas.repec.org/p/nbr/nberwo/15029.html.

Federico Etro. The theory of endogenous market structures: A survey. Working Papers 11, Department of Economics, University of Venice "Ca' Foscari", July 2012. URL http://ideas.repec.org/p/ven/wpaper/2012_11.html.

Jordi Galí and Fabrizio Zilibotti. Endogenous growth and poverty traps in a cournotian model. *Annales d'Economie et de Statistique*, (37-38):197–213, 1995. URL http://EconPapers.repec.org/RePEc:adr:anecst:y:1995:i:37-38:p:10.

Nir Jaimovich and Max Floetotto. Firm dynamics, markup variations, and the business cycle. *Journal of Monetary Economics*, 55(7):1238–1252, 2008. URL http://EconPapers.repec.org/RePEc:eee:moneco:v:55:y:2008:i:7:p:1238-1252.

Larry E. Jones, Rodolfo E. Manuelli, and Ennio Stacchetti. Technology (and policy) shocks in models of endogenous growth. Staff Report 281, Federal Reserve Bank of Minneapolis, 1 2000. URL http://ideas.repec.org/p/fip/fedmsr/281.html.

Robert G. King and Sérgio Rebelo. Business cycles with endogenous growth. Unpublished manuscript, 2 1986.

Lilia Maliar and Serguei Maliar. Endogenous growth and endogenous business cycles. *Macroeconomic Dynamics*, 8(05):559–581, November 2004. URL http://ideas.repec.org/a/cup/macdyn/v8y2004i05p559-581_04.html.

María Paz Moral Zuazo and Ilaski Barañano. Output dynamics in an endogenous growth model. BILTOKI 2003-10, Universidad del País Vasco - Departamento de Economía Aplicada III (Econometría y Estadística), 2003. URL http://EconPapers.repec.org/RePEc:ehu:biltok:200310.

Elvan Ozlu. Aggregate economic fluctuations in endogenous growth models. *Journal of Macroeconomics*, 18(1):27–47, 1996. URL http://ideas.repec.org/a/eee/jmacro/v18y1996i1p27-47.html.

Alessandra Pelloni. Nominal shocks, endogenous growth and the business cycle. *Economic Journal*, 107(441):467–74, March 1997. URL http://ideas.repec.org/a/ecj/econjl/v107y1997i441p467-74.html.

Kerk L. Phillips and Jeff Wrase. Is schumpeterian 'creative destruction' a plausible source of endogenous real business cycle shocks? *Journal of Economic Dynamics and Control*, 30(11):1885–1913, November 2006. URL http://ideas.repec.org/a/eee/dyncon/v30y2006i11p1885-1913.html.

Lukas Schmid and Howard Kung. Innovation, growth, and asset pricing. 2011 Meeting Papers 1325, Society for Economic Dynamics, 1 2011. URL http://ideas.repec.org/p/red/sed011/1325.html.

George W Stadler. Business cycle models with endogenous technology. *American Economic Review*, 80(4):763–78, 1990. URL http://EconPapers.repec.org/RePEc:aea:aecrev:v:80:y:1990:i:4:p:763-78.

Yi Wen. Why does consumption lead the business cycle? Working Papers 01-08, Cornell University, Center for Analytic Economics, 2 2001. URL http://ideas.repec.org/p/ecl/corcae/01-08.html.