

THE OPPORTUNITY COST HYPOTHESIS AND THE CYCLICAL BEHAVIOUR OF RESEARCH SPENDING

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ABSTRACT. Schumpeter's memorable aphorism of "creative destruction" has spawned numerous important contributions in the study of economic growth but there has been substantially less attention paid to his hypothesis that R&D expenditure should display counter-cyclical features. I outline a dynamic stochastic general equilibrium model in which research expenditure drives innovation and growth, and attempt to reconcile empirical results suggesting both pro-cyclical of R&D and research expenditure smoothing with the response of those variables in the theoretical model. It is then extended to allow for an endogenous market structure so that the behaviour of research spending at three levels of aggregation can be analysed. The model generates a behaviour of R&D spending that is broadly consistent with empirical evidence regarding its pro-cyclical and outlines the mechanisms through which that occurs. I then briefly discuss the effect of business cycle fluctuations on the time paths of aggregate productivity and aggregate output in each of the models analysed.

Keywords: research & development, endogenous growth, cycles.

JEL Classification: E32, E44, O31, O32, O40

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1. INTRODUCTION

A famous hypothesis originally formulated by Schumpeter (1939) makes the argument that recessions are ideal opportunities to invest in research that would yield new growth and profit generating technologies. The logic is simple: idle resources in a downturn can be reallocated to inventive activity at a lower cost than at any other point in the cycle. Thus, downturns provide excellent opportunities to invest resources in productivity enhancing technologies.

Despite this, both empirical and theoretical contributions have since supplied ample evidence to suggest R&D expenditure or indeed effort¹ very clearly displays pro-cyclical behaviour. This is entirely at odds with the assumption that a lower opportunity cost during recessions should incentivise efforts to discover new technologies which could then be profitably exploited in periods of expansion.

This paper first briefly summarises a few key empirical findings that show pro-cyclical behaviour of research spending in both aggregate and disaggregated data, as well as those theoretical contributions that attempt to explain the reasons why it fails to corroborate the Schumpeterian hypothesis. It then develops a simple Schumpeterian model of endogenous growth² embedded in a standard real business cycle framework in which entrants rely on venture capital or debt to finance research expenditure. The trade-off between production and research only works through general equilibrium channels in this setting, so the model is extended to allow for incumbents to undertake research expenditure. Rather than generating new innovations, the latter agents are assumed to engage in imitation of new technologies, which in turn leads to an endogenous market structure. This is useful as it allows for a characterisation of research expenditure for the entire economy: total aggregate research spending, research spending by all entrants and research spending by individual firms. Disaggregation allows for a characterisation of the problem each individual firm faces, which in effect is the original trade-off envisioned by Schumpeter.

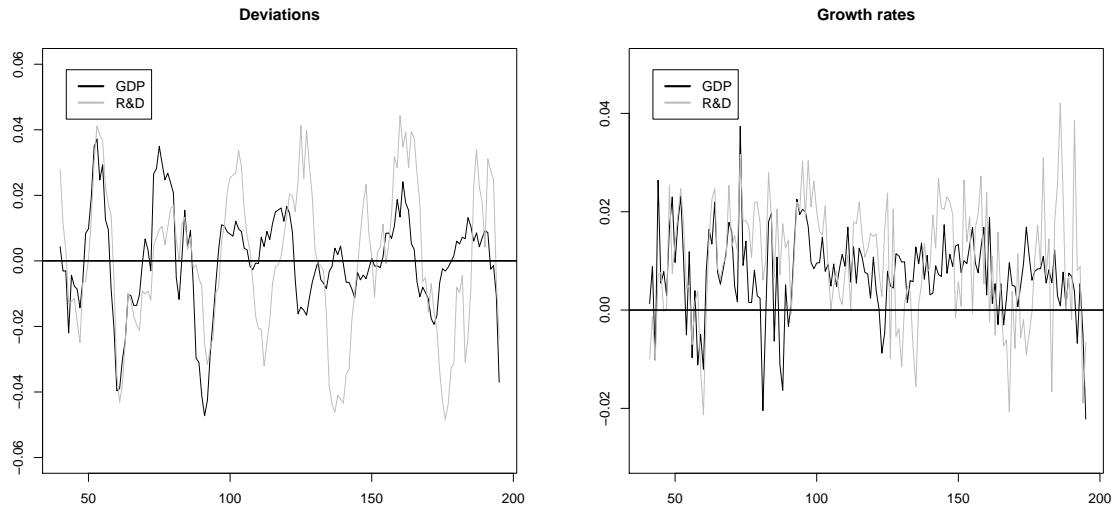
¹Measured as the fraction of labour resources devoted to research activity.

²The literature refers to these models as 'quality ladders'. In previous work, I discuss three competing approaches that attempt to integrate these models in a real business cycle framework.

2. LITERATURE

2.1. Empirical Evidence. Figure 1 establishes a *prima facie* argument for what appears to be a positive correlation between real GDP and aggregate research expenditure. Indeed, while the graph on the right side looks at the growth rates for both of these variables in the sampled period, the one of the left shows their deviations from the trend, calculated using a Hodrick-Prescott filter. The same conclusion is immediately apparent in both graphs, and indeed correlation between deviations in aggregate R&D and in real GDP rounds up to 45.2%, while the equivalent for the growth rates in both variables is 33.3%. Counter-cyclical behaviour in aggregate research spending is simply not a feature of the data.

FIGURE 1. Relationship between aggregate R&D and RGDP



More importantly, though, recent research (Barlevy (2007), Comin and Gertler (2006)³, Fatás (2000), Ouyang (2010b), Ouyang (2010a), Wälde and Woitek (2004)) has collected significant evidence contra the Schumpeterian hypothesis, instead postulating that R&D is strongly pro-cyclical at the aggregate and industry level. One of the explanations put forward in order to explain the cyclical profile of research spending that it is technology shocks, by raising the value of innovations through increased productivity of ideas, driving

³The latter argue that this is evident not only in higher frequencies but also medium-term frequencies, as low medium-term waves of economic growth tend to reflect on relatively low R&D effort.

increases in R&D. This explanation is based on the cyclical behaviour of profits: should these display a strongly pro-cyclical bias, it will likely force research spending to follow the same pattern despite the counter-cyclical bias implied by the opportunity cost hypothesis.

Similarly, changes to aggregate of industry-wide demand can have similar effects: by affecting the cyclical pattern of profits, it similarly offsets the counter-cyclical opportunity cost effect by changing the value of firms and, therefore, the optimal amounts of research. This type of explanation is not based on a departure from the opportunity cost hypothesis, but of "dynamic externalities" (Barlevy (2007)) that cause entrepreneur to behave in a shortsighted fashion or demand-side variability as is argued in Geroski and Walters (1995). The argument for a demand-pull explanation of research investment relies on upward shifts in demand increasing the expected profitability of innovations and, hence, stimulating investment in research⁴.

A competing theory to both hypotheses based on aggregate shocks having implications for the volatility of profits, and, hence, of firms' valuations, is the notion that despite there being a strong incentive to engage in research spending during downturns, there may be a scarcity of available funds during these periods. That is: if access to credit or financial markets is restricted during periods of economic contraction, it follows that even though firms and entrepreneurs would very much like to expand the rubric of research expenditure at the expense of resources devoted to production, the reality is such that they cannot obtain the necessary funding in order to do so.

Hence, some authors maintain that the opportunity cost hypothesis does not hold because of the restricted availability of credit. Aghion et al. (2008), argue that absent credit constraints, empirical evidence from France suggests private R&D is actually counter-cyclical and that, even allowing for credit constraints, there is an asymmetric response of R&D to economic conditions, in that firms decrease expenditure sharply during recessions as credit tightens but do not increase it accordingly during periods of recovery. This insight is corroborated in Aghion et al. (2005) using a simple model that implies an increase in the pro-cyclical behaviour of R&D when there are constraints on the availability of credit. Additional research by Mancusi and Vezzulli (2010), present evidence that

⁴Arguments along those lines can be extended to any type of shock that increases the equilibrium or steady-state value of the firm: even if the substitution effect decreases R&D expenditure, the increase in firm value more than compensates for it and it may well be expected to increase, thus seemingly invalidating the opportunity cost hypothesis.

credit constraints have "a significantly negative effect on the probability to set up R&D activities".

In the second chapter of this thesis, I looked at the relationship between research expenditure and output at the industry and firm levels and found strong evidence in favour of the hypothesis that firms engage in R&D smoothing. That hypothesis can be summarised as follows: research spending moves in lock step with the cycle, along all other expenditure variables, but its share of final expenditure and its share of total investment⁵ fall during output expansions. In short, at the industry and firm level, changes in output trigger a less than proportional response of R&D expenditure. The implication of this is that while real expenditure moves with the cycle, its share is strongly countercyclical, suggesting that the opportunity cost hypothesis is true in a more limited sense. Further, the claim that financial constraints are important in explaining the pro-cyclical behaviour of R&D are not reproduced in the data I analysed. In other words, the relevant measures behave identically whether or not firms show signs of having significant restrictions on their access to funding: research spending is pro-cyclical and the shares behave counter-cyclically.

2.2. Theoretical Literature. The behaviour of R&D is likely to be very sensitive to general equilibrium dynamics, in particular when it comes to the decisions taken by entrepreneurs that are developing disruptive technologies which will be embodied in new firms; this class of agents is unlikely to respond directly to temporary fluctuations in demand because they don't face a trade-off between production and R&D. Therefore, the results of partial equilibrium models, such as the aforementioned Aghion et al. (2005), may not be general because they abstract from these economy-wide effects. In order to do so, other authors have attempted to incorporate models of cycles with endogenous adoption of new technologies. I review some of the most significant contributions, as well as the results which more directly concern the question of the cyclicity of R&D.

Several authors have recently attempted to integrate Schumpeterian growth dynamics with the standard RBC model, but as early as Pelloni (1997) , there had been attempts to reconcile business cycle dynamics with endogenous acquisition of knowledge, chiefly through learning by doing or other human capital investments. Contributions in this vein have been revised to accommodate Schumpeterian features, as in Maliar and Maliar (2004)

⁵Total investment is defined as the sum of research and capital formation expenditure.

, but these authors find that while their model predicts highly pro-cyclical innovations, their cycle-generating mechanism (the shifting of resources between R&D and production) implies that R&D is counter-cyclical, much in the vein of the opportunity cost hypothesis. This is at variance with what the empirical evidence mentioned previously suggests. A variety of theoretical models, such as Bean (1990), Canton (2002), and Ozlu (1996), corroborate the counter-cyclical behaviour of human capital accumulation, which in turn agrees with empirical evidence mentioned by these authors. The usefulness of models that use human capital as a proxy for innovative activity and as a guide to how R&D responds to output variability is, however, compromised by the fact that it displays a counter-cyclical behaviour, a variable that has been shown to have, at the very least, moderately pro-cyclical behaviour.

To make this point clearer, in Maliar and Maliar (2004), the authors outline what is essentially a model of human capital accumulation in which investments in human capital are thought of as a form of R&D spending⁶. That being the case, it is hard to reconcile the evidence of pro-cyclical R&D with counter-cyclical human capital investments⁷. Thus, the clarity of models that conflate the two is hampered significantly while models that rely on some form of human capital investment or learning-by-doing do not yield meaningful predictions about the behaviour of R&D over the cycle.

Hence, a theoretical model attempting to provide a rationale for the behaviour of research expenditure time series must explicitly describe decisions to invest in innovative activity by firms, consumers or a social planner. There is significant variety in these, meaning that their predictions might not be comparable across models, but all provide reasonable and workable approximations to the task of exploring the role of economic cycles in a growth model with *creative destruction*.

Phillips and Wrage (2006), develop a model in which the process of 'creative destruction' is not a reasonable source of technology fluctuations, but they nevertheless succeed in developing an approach that allows for an analysis of how exogenous technology shocks affect the cyclical behaviour of expenditure on innovation; an approach which is partially replicated in these pages. Other authors, such as Fatás (2000), developed models in which

⁶The authors explicitly describe it as an R&D variable.

⁷Sakellaris and Spilimbergo (2000) present evidence suggesting that human capital is in fact counter-cyclical for OECD countries while credit constraints seem to account for pro-cyclical behaviour for non-OECD countries.

variable aggregate demand, via shocks to employment, is the main source of fluctuations. This particular model highlights the effect of fluctuations in demand in the profitability of innovations, which in turn affect firms optimal choices of research effort and are the reason why R&D exhibits pro-cyclical behaviour.

Comin and Gertler (2006), argue that are medium-term cycles in a number of relevant macro variables and, significantly for our discussion, that R&D exhibits pro-cyclical behaviour not only at higher frequencies but also in the medium term. That is, apart from a positive correlation with fluctuations in output in the short-run, R&D seems to follow what the authors claim to be medium-term (32 to 200 quarters) cycles. They rely, however, in an exogenous shock in households labour-leisure choice following evidence⁸ suggesting that this is the most important source of cyclical variation. Even though their model yields a wide variety of predictions consistent with the data, the authors argue that inclusion of a process of diffusion of technology has scope to greatly improve the performance of the model provided technology adoption is pro-cyclical. This general framework, expanded in Comin et al. (2009), it is a significant improvement on preceding work, and is very similar in structure to the approach pursued herein. This approach is again closely linked to work developed by Barrau (2010) and Barrau (2008), the latter of which includes one of the first attempts in the literature to incorporate credit restrictions in a model with both endogenous growth and cycles.

A model which deals explicitly with the question of the timing, i.e, the cyclical of research spending, is Barlevy (2004). In this model, intermediate good producers face a dynamic trade-off in which reducing the amount of labour allocated to production directly contributes to an increase in labour productivity in future periods. He then concludes that while R&D ought to be counter-cyclical in the socially optimal policy rule, the decentralised equilibrium generates pro-cyclical research spending when fixed costs are considered.

Despite most models generating pro-cyclical R&D behaviour, with the obvious exception of those which rely on human capital as a proxy for research expenditure, they don't address important questions such as whether research expenditure is 'smoothed' out by firms experiencing output volatility from exogenous shocks and whether the more limited

⁸See Hall (1997), for more detail.

version of the opportunity cost hypothesis holds. Further, the pro-cyclicality of the number of firms implies that while aggregate expenditure in research may be pro-cyclical, the behaviour of individual firms is impossible to ascertain from models in which there is no endogenous market structure. Finally, explanations for the observed statistical behaviour of R&D that rely on restrictions to financing, whether through equity or debt, remain under-explored.

2.3. Summary. Addressing all the concerns raised in the preceding section, informed by the problems and limitations of existing theoretical models, involves creating a simple, stripped down model that combines growth, fluctuations and financial constraints. I propose several variants of a real business cycle model in which labour productivity growth is endogenously determined and in which entrepreneurs face constraints in their access to resources with which research efforts are to be financed.

The first of these models follows the basic underlying structure of an RBC model with no frictions apart from a financing constraint. There are two types of household, patient and impatient, and the latter engage in the creation of new firms by developing better technologies that enable them to replace current incumbents. Because they don't earn a wage income, both their consumption of the final good and research expenditure are financed by agreeing upon relinquishing a fraction of the future value of any firms created through their innovative activity, a model I describe as being akin to that of 'venture capital' financing. This framework, however, may not entirely capture the way financial frictions restrict the ability of entrepreneurs to engage in innovative activity. In particular, Adelino et al. (2013) find that collateral, in the form of housing, plays an important role in fomenting entrepreneurship. With that in mind, I change the economic environment appropriately so as to allow entrepreneurs to amass physical capital which they can then use as collateral to borrow against. This is important because these two channels may entail different responses from research spending to various exogenous shocks, in particular because in the latter case the response of the level of capital impacts research expenditure decisions through the entrepreneur's budget and financial constraints.

A potential issue with taking this approach is that in models where entrepreneurs are responsible for all of the research expenditure, these agents may not face a trade-off between production and R&D. In particular, the Schumpeterian argument that firms will shift resources away from production to engage in research spending when demand for

output is lower may not apply in this context because entrepreneurs will likely reduce expenditure when output is lower, leading research expenditure to behave pro-cyclically. To examine the way firms would respond to fluctuations in output brought about by exogenous shocks, I allow incumbents to engage in research expenditure that allows them to, if successful, copy newly developed technologies to remain in the market. Rather than allowing incumbents to develop their own innovations, by allowing firms to copy new technologies, the assumption of a monopolistic market structure is no longer required, and market structure will then be determined endogenously.

The advantages of this structure are twofold: firm's optimal research expenditures can be observed directly, and rather than look at R&D by industry, each firm's individual decisions are equally determined by the model. Indeed, the cyclical profile of industry research expenditure may not be indicative of individual firms' optimal response because of aggregation: the entry of new firms in periods of higher than normal output may lead to pro-cyclical research spending even if individual firms engage in less innovative activity. Again, to account for the different responses generated by different financing structures, two versions of this model are simulated, one in which entrepreneurs resort to venture capital financing and another in which they can use capital as collateral against which to borrow.

The cyclicity of research expenditure has significant implications for the debate over the costs of economic fluctuations, and depending on whether it moves along with the cycle or against it, the policy conclusions vary in important ways. Pro-cyclical R&D, under certain conditions, could imply that deep recessions could have a very large negative impact on productivity growth, thereby reducing potential output in future periods. On the opposite end, counter-cyclical spending would imply that recessions have a silver lining by spurring on innovations which would increase future productivity and output. The models proposed in this paper allow us to examine how temporary shocks affect the cyclical behaviour of research expenditure and provide a framework in which future research on the cost of the business cycle identified in Barlevy (2004) can be based.

3. MODEL

The presentation of the four variants of the model are structured as follows. First I introduce the elements that are common across model specifications in section (3.1), titled 'standard model' precisely because it introduces the simplest and broadest version, upon

which all subsequent modifications are imposed. In this initial version, entrepreneurs finance research expenditures by alienating a fraction of future companies *a priori*, with their participation in them being sold off when a new firm is created. This approach is similar to that followed in Keuschnigg (2004), where entrepreneurs and venture capitalists bargain over the share allocations of successfully developed new enterprises, but rather than explicitly model that process of negotiation, I assume that the share desired by the patient households is a stochastically perturbed parameter. I follow this strategy because that allows me to calibrated the model to empirically observed venture capitalist participations in publicly floated companies⁹ and because this parameter can then be used to model an exogenous tightening of the availability of financing, which in this case takes the form of seed capital made available by the patient households to entrepreneurs.

In section (3.2), I introduce the flexible mark up version of the model in which market structure is endogenous, as determined by incumbents' imitation efforts. Entrepreneurs continue to be financed through the alienation of future equity participations in the new company, while incumbents can finance imitation expenditure from operational profits.

Finally, I then modify the entrepreneur financing mechanism in both models to allow impatient households to borrow against collateral in section (3.3). This modification is motivated by evidence suggesting that collateral backed debt is an important mechanism for new business creation (Adelino et al. (2013)), especially given the significantly less important role that venture capital plays in the development of new businesses. This is accomplished by allowing entrepreneurs to accumulate physical capital, which they can then use as collateral in securing loans from the patient households. This choice of financial friction was first proposed by Kiyotaki and Moore (1997), and the approach taken here closely follows the specification used in Brzoza-Brzezina et al. (2013).

Unlike these authors, however, I abstract from fully describing the financial or banking sector. In this class of models, the banking sector can access financing at the risk free interest rate set by the monetary authority and charges a premium over this rate. The absence of a monetary authority in this setting means I can abstract from the role played by the banking system and, given the differing rates of time preference between households, model patient households as requiring the postage of collateral. The loan-to-value ratio is

⁹In Keuschnigg (2004) this would not be possible as under most specifications the Nash bargaining outcome would lead to an equal share distribution.

then modelled as a stochastically perturbed parameter that can be interpreted as the exogenous availability of credit. Finally, this framework is identical to the Brzoza-Brzezina et al. (2013) formulation, without banks and admitting only financial intermediaries.

3.1. Standard model.¹⁰ The basic outline of the model is a discrete time version of a textbook quality ladder model, in which investments in research and development generate an endogenous probability of an increase in the sector's quality level. Aggregating across all the sectors in the economy yields the endogenous growth rate for aggregate labour productivity. The remaining structure of the model is identical to a standard RBC model with exogenous price mark-ups generated by the Dixit-Stiglitz structure of the final good aggregator. Entrepreneurs are constrained in their ability to invest in R&D by the exogenous fraction of equity consumers are willing to purchase in any new firm (their equity share) in what could be described as a venture capital financing structure.

3.1.1. Households. There is a unit measure of patient households, the representative of which faces a standard problem of maximising a time separable utility function over consumption of a composite final good and hours worked. Supplied labour is hired out by intermediate goods firms at the wage rate w_t . In addition to this, the representative consumer must also choose her preferred amount of investment or seed money, $B_t(\iota)$, in the entrepreneurial households, as well as physical capital that is rented out to intermediate goods firms. In the event a new firm is created, each household then purchases a fraction s outright from entrepreneurs, so that all firms are eventually fully owned by the patient households. This means firm profits¹¹ (Σ_t) are paid directly to these agents. The optimisation problem for the ι^{th} patient household can be written as follows:

$$(1) \quad \begin{aligned} \max_{\{c_t(\iota), h_t(\iota), B_t(\iota), k_t(\iota)\}_{t=0}^{\infty}} U &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ v_t \log c_t(\iota) - \varphi \frac{h_t(\iota)^{1+\psi}}{1+\psi} \right\} \\ \text{s.t.} \quad w_t h_t(\iota) + s(\iota) \Sigma_t(\iota) + R_t^k k_t(\iota) + R_t^b B_t(\iota) &= \\ &= c_t(\iota) + k_{t+1}(\iota) + B_{t+1}(\iota) + s(\iota) \int_0^1 (1 - m_{t-1}) \eta(z_{t-1}^E(e)) V_t^j(q_i(e)) de \end{aligned}$$

¹⁰In sections (4.1) and (5), this model will be referred to as Standard RBC VC.

¹¹These are defined as operational profits, π_t , net of research expenditures, z_t^j . In this setting, because firms do not engage in research spending of any kind, $\pi = \Sigma$. This will be relaxed further ahead, as outlined earlier.

The last term on the right hand side of the household's budget constraint gives us the cost of acquiring a fraction of all new firms created throughout the economy (aggregating over all e sectors) at time t . It explicitly depends on the probability of a successful innovation, $\eta(\cdot)$, because consumers plan expenditures at time 0, before the realisation takes place at time t . Aggregate demand shocks are modelled as preference shifters that increase the period return to consumption over leisure and follow the approach used in Ireland (2004) and obey the following law of motion:

$$(2) \quad \log(v_t) = \rho^v \log(v_{t-1}) + \varepsilon_t^v$$

Entrepreneurial or impatient households are also scaled to a unit measure aggregated over all the sectors e in the economy, and the representative agent in this setting faces a very different problem. She does not earn any labour income as her only activity concerns the production of new ideas that can be transformed into successful innovations and, consequently, firms. In order to finance research expenditure, as well as consumption, entrepreneurs must successfully secure investment from the patient households that takes the form of seed money $L_t(e)$. The cost of doing research and invention probabilities are introduced here to complete the representative impatient household's budget constraint but warrant a closer and more systematic exposition which will be left to the section in which research and technological progress are discussed in more detail.

$$(3) \quad \begin{aligned} \max_{\{c_t^E(e), L_t(e), z_t^E(e)\}_{t=0}^{\infty}} U^e &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_e^t \{\log c_t^E(e)\} \\ \text{s.t.} \quad c_t^E(e) + z_t^E(e)q_i(e) + R_t^b L_{t-1}(e) &= L_t(e) + \eta(z_{t-1}^E(e))V_t^j(q_i(e)) \\ R_{t+1}^b L_t(e) &\leq m_t \eta(z_t^E(e)) \mathbb{E}_t[V_{t+1}^j(q_{i+1}(e))] \end{aligned}$$

Absent the last constraint in the impatient household's optimisation problem, entrepreneurs would always successfully fully smooth consumption and invest in research so that the cost of allocating an additional unit of the final good to research expenditure would equate the expected increase in the probability a firm of a given value is created. Patient households, however, are wary of investing too large a stake in potential innovations and therefore restrict the amount they are willing to lend entrepreneurs. In conjunction with a higher degree of impatience by these households, this introduces a wedge between the returns to engaging in innovative activity and the rate at which impatient households

must pay to access these resources. Rather than an interest rate as such, R^b represents the internal rate of return on a seed money investment in a budding entrepreneur.

Introducing financing constraints allows us to examine possible ways in which access to resources can affect entrepreneur's ability to engage in inventive activity, either through exogenous shocks to the patient household's willingness to finance these expenditures or through a financial amplification mechanism, in which exogenous shocks to productivity or demand may affect lending and then feedback onto aggregate economic activity.

3.1.2. Production. There is a unit measure of sectors in this economy. In each sector e , there is a single intermediate producer which competes with all other sectors. There is a single final good and a single capital good, both of which are produced by firms acting in perfectly competitive markets. Capital is rented out by households to intermediate goods firms, which use it to produce a sectoral output that is in turn used in the production of final output. Final output is then used for consumption (of both sets of households), capital production and research efforts.

3.1.3. Capital producers. Capital is produced competitively and the absence of adjustment costs means the price of capital is the same as the price of final output. The production technology yields the standard law of motion for capital:

$$(4) \quad k_{t+1} = (1 - \delta)k_t + i_t$$

3.1.4. Final good. Final output is a composite of a unit mass of sectoral goods, each at potentially different stages of technological sophistication. Each sector produces a single, undifferentiated good:

$$(5) \quad Y_t = \left[\int_0^1 \left(\omega_t(e) y_t(e)^{\frac{\mu-1}{\mu}} \right) de \right]^{\frac{\mu}{\mu-1}}$$

Where $\omega_t(e)$ is a weight assigned to each sectoral quality level that takes the form¹²:

$$(6) \quad \omega_t(e) = \frac{u_t q_t(e)}{\left(\int_0^1 u_t q_t(e) de \right)^{\frac{1+\alpha(\mu-1)}{\mu}}}$$

Aggregate productivity has two components, the endogenous element arising from the quality increasing research efforts of innovators and an error term that introduces random fluctuations in labour productivity. Both enter the aggregate production function through

¹²This weight ensures aggregate output is linear in the average quality across sectoral goods.

the weight function ω defined above in (6). The random component of aggregate labour productivity obeys the following law of motion:

$$\log(u_t) = \rho_u \log(u_{t-1}) + \varepsilon_t^u$$

The production technology for each sectoral monopolist is given by the standard Cobb-Douglas production function:

$$(7) \quad y_t(e) = [k_t(e)]^\alpha [l_t(e)]^{1-\alpha}$$

Each firm must solve a two-step optimisation problem. First, it must choose the level of inputs that minimises costs relative to a fixed revenue:

$$(8) \quad \begin{aligned} \min_{k_t(e), l_t(e)} & MC_t(e)y_t(e) - k_t(e)(R_t^k - 1 + \delta) - l_t(e)w_t \\ \text{s.t.} & MC_t(e)y_t(e) \leq E \end{aligned}$$

Second, it must choose an optimal mark-up over marginal cost, where demand for the sectoral good is derived from the final producer's demand for each intermediate input. This price mark-up takes the familiar form:

$$(9) \quad p_t(e) = \mu MC_t(e)$$

From which the profit function can be easily derived:

$$(10) \quad \pi_t(e) = \frac{\mu - 1}{\mu} Y_t$$

Solving for these gives us the optimal mark-up in equation (9) as well as the demands for both inputs. I abstract from frictions in pricing and assume that the monopolist in every sector can reset the price optimally every period, which means the price of the final good, $P_t = 1$, can be used as the *numéraire* in this economy.

3.1.5. Research. Unlike standard real business cycle models, in which the growth rate of output is taken as an exogenous quantity, this model explicitly captures the process through which technological progress occurs by modelling the relationship between the flow of innovations and research expenditure.

At any time t we can define the economy-wide average quality level as Q_t , which can be found by simply aggregating over the quality level in all sectors. This is important because the degree of technological sophistication may be quite different across sectors,

and aggregating them into a single average measure allows us to abstract completely from these differences. That measure is defined quite simply as follows:

$$(11) \quad Q_t = \int_0^1 q_t(e) de$$

Innovation is said to occur in a specific sector e when an challenger is successful in developing a better quality version of the intermediate good produced in that sector. The probability that any given entrepreneur is successful is given by:

$$(12) \quad \text{Prob}(\text{entrepreneur } e \text{ is successful}) = \eta(z_t^E(e))$$

This probability depends on the quality-adjusted level of expenditure by innovators and it is assumed to display diminishing marginal returns. Each sector's quality ladder evolves according to the following law of motion:

$$(13) \quad q_{i+1}(e) = \lambda q_i(e)$$

Interpreting this is straightforward: when a successful innovation occurs, there is a jump in quality equal to $\lambda - 1$. To ensure that there is no limit pricing, the price markup charged by the firm holding the new technology must be lower than the productivity increase engendered by the quality improvement. Because the displaced incumbent can always sell units produced at the previous quality level up to a fraction of the innovator's marginal cost, this condition is necessary in order to ensure that incumbents cannot profitably sell an earlier vintage of the intermediate good. This is, in turn, equivalent to imposing a lower bound on each step of the quality ladder:

$$(14) \quad \mu^{\frac{1}{1-\alpha}} < \lambda$$

From here, we can move on to the law of motion for the average quality level in the economy. The law of motion for the entire economy follows from the law of large numbers and under the assumption that probabilities of innovation are independent of the quality level in other sectors:

$$(15) \quad \mathbb{E}_t[Q_{t+1}] = \left(\int_0^1 \eta(z_t^E(e)) de \right) \lambda Q_t + \left(1 - \int_0^1 \eta(z_t^e(m)) de \right) Q_t$$

The logic here is straightforward: the expected level of quality in period $t + 1$ is given by the sum of the probability that any of the entrepreneurs is successful in developing a

new innovation, which increases the average quality level by λ , and the probability that none of them achieves that aim.

Research is conducted by entrepreneurs, who purchase units of the final good and transform it into research expenditure, the cost of which is assumed to depend linearly on the quality level. This assumption reflects the idea that as technology becomes more sophisticated, generating new innovations becomes increasingly harder. It is also essential to ensure that expenditure and production will grow at the same rate along the balanced growth path. This cost of doing research was introduced in the impatient household's budget constraint and simply takes the form:

$$(16) \quad z_t^E(e)q_t(e)$$

All of this implies that the value of an existing firm must obey the following law of motion:

$$(17) \quad V_t^j(q_i(e)) = \pi_t^j(q_t(e)) + (1 - \eta(z_t^E(e))) \mathbb{E}_t[\Lambda_{t,t+1} V_{t+1}^j(q_i(e))]$$

Where the stochastic discount factor for the firm comes from the optimisation problem for the patient household:

$$(18) \quad \Lambda_{t,t+1} = \beta \mathbb{E}_t \left[\frac{v_{t+1}}{v_t} \frac{c_t}{c_{t+1}} \right]$$

From the impatient household's optimisation problem, the first order condition with respect to $z_t^E(e)$ results in:

$$(19) \quad -q_t(e) + \frac{d\eta(z_t^E(e))}{dz_t^E(e)} \mathbb{E}_t [(\Lambda_{t,t+1}^E + m_t \Omega_t) V_{t+1}^j(q_{i+1}(e), n_{t+1}(e))] = 0$$

Where the discount factor is defined as in (18), but for the impatient households, and Ω_t is the Lagrangian multiplier associated with the financing constraint for this class of household.

Incumbents are assumed not to have access to the innovation generating technology. In standard models, this assumption is innocuous because the Arrow replacement effect ensures that incumbents face much smaller marginal gains from engaging in research activity and, therefore, entrants can always outbid established firms and 'price' them out of the market. In this setting, however, entrepreneurs are constrained while firms can invest out of the profits they generate, which implies the replacement effect might not

hold under certain conditions. In order to ensure this is the case, along the balanced growth path the following condition must hold:

$$(20) \quad \mu^{\frac{1}{1-\alpha}} < \lambda < \frac{\beta}{\beta - \beta_E} \cdot \frac{1}{1-m}$$

Which implies that the jump in quality, apart from being bounded from below as per equation (14), it must also be bounded above so as to ensure that the Arrow effect does not hold. The intuition for this result is simple. If $m = 1$, this implies a negative quality jump, which clearly cannot be true. More clearly, this means that the constraint is not binding and, therefore, entrepreneurs and incumbents are in equal footing, implying that the former will always outbid the latter. For $0 < m < 1$, the jump in quality must be very large¹³ for the Arrow replacement effect not to hold. In particular, it must be large enough that entrepreneurs are unable to finance additional spending at the margin, while the incumbent would always be able to finance more research spending. Quantitatively, however, this condition is easily satisfied for any plausible calibration of the model.

3.1.6. *Resource constraint.* Aggregating the budget constraints of both types of household and assuming that the financing constraint in the entrepreneurial household is satisfied with equality, the following aggregate resource constraint results:

$$(21) \quad Y_t = C_t + C_t^E + I_t + Z_t$$

3.2. **Endogenous market structure.** ¹⁴ In order to examine how firms respond to the research/production trade-off implied by the opportunity cost hypothesis, firms must be allowed to conduct research expenditure, given that entrepreneurs are uniquely concerned with the generation of new innovations. To achieve this, a few changes to the model discussed in the previous section are introduced. Instead of the benchmark Dixit-Stiglitz model of imperfect competition, all firms in each sector are assumed to produce and sell an identical good and to compete à lá Cournot. The immediate result is that market power comes not from selling a different variety of the good over which the firm holds a patent but from the fact that there is a finite number of firms in the market for that specific sectoral good at each point in time. One implication is that, in the limit, each good's price could, conceivably, be equal to the price that would prevail in a competitive

¹³The choice of values for β and β_E in the calibration of the model would then imply quality jumps that are incompatible with the existence of a balanced growth path solution.

¹⁴In sections (4.1) and (5), this model will be referred to as Endogenous mark-up (End MkUp) VC.

setting should the number of firms approach infinity. Thus, the model allows for a full range of potential market structures, from monopoly to perfect competition.

Tied to this initial departure is the process through which innovation and technology acquisition occurs. Condition (20) ensures that the Arrow replacement effect will hold, absent any specific advantages that incumbents may possess in the innovative process, meaning that challengers will always outbid the former when competing for resources to devote to innovation. Technological progress occurs when an entrant successfully improves on the current quality level of the good being produced.

Incumbents are not, however, helpless in the face of creative destruction: in the absence of well defined intellectual property rights, as soon as a new technology is developed, they can attempt to reverse engineer it and reenter the market in order to compete on an equal footing with the successful entrepreneur. This process is assumed to be instantaneous and occurs right before production takes place, meaning that any and all incumbents that are able to replicate the new technology at the new quality level will remain in the market, while those who fail to do so lose their market position and see firm value liquidated.

Entrants are then assumed not to be able to imitate a new technology developed¹⁵ by one of their own. In short, only incumbents, who possess the technology that has been just displaced, are able to reproduce the technology just discovered by an entrepreneur. This assumption resonates with the widely held perception that the majority of innovations are come by through the efforts of innovative outsiders, rather than well established firms attempting to cement their market position. It also validates the perception that despite not being as innovative, firms can remain at the technological frontier if they are sufficiently adept at adopting new innovations and bringing to market equivalent products, thereby competing with innovators on an equal footing.

3.2.1. Households. The infinite horizon optimisation problems faced by each household are identical in this section, the only difference emerging from the fact that firm value $V(\cdot)$ is now also a function of the number of firms as well as the quality level.

¹⁵Indeed, while there is a continuum of these, by virtue of there being a continuum of entrepreneurial households, I assume in the equilibrium section that they are all identical and, therefore, behave exactly like a representative entrepreneur would, rendering the issue of copying one's own invention irrelevant. Nevertheless, the assumption that copying is impossible unless in possession of the technology that has just been displaced avoids that potentially complicated result.

3.2.2. *Production.* In this setting, the economy is still comprised of a unit measure of sectors, each of which is populated by a number $n_t(e)$ of intermediate producers that compete *à la* Cournot to produce an identical sectoral good. Final and capital good production are identical to those outlined in the previous section.

3.2.3. *Final good.* The final good aggregator suffers minor changes, however. Because each sector produces a single, undifferentiated good and there is a finite number of producers, $n_t(e)$, in each of them, final output is defined as:

$$(22) \quad Y_t \equiv \left[\int_0^1 \left(\omega_t(e) \int_0^{n_t(e)} y_t^j(e)^{\frac{\mu-1}{\mu}} dj \right) de \right]^{\frac{\mu}{\mu-1}}$$

Where again $\omega_t(e)$ takes the form of (6). Equally, the aggregate productivity shock u_t is modelled as in the previous section, while the intermediate good production function for firm j is defined as in (7).

The assumption of competition in quantity, *à la* Cournot, yields a price mark-up that is a function of the price elasticity of the sectoral output as well as the number of firms in each market. This closely follows the approaches developed in Galí and Zilibotti (1995),¹⁶:

$$(23) \quad p_t(e) = \frac{\mu n_t(e)}{\mu n_t(e) - 1} MC_t^j(e)$$

The implication is clear: as the number of firms in sector e increases, it tends to the competitive outcome of a zero mark-up over marginal cost. Under the assumption that sectoral output is evenly divided between all market participants, we can write each individual firm's operational profit¹⁷ function as:

$$(24) \quad \pi_t^j(e) = \frac{1}{\mu n_t(e)} \frac{Y_t}{n_t(e)}$$

Choice of optimal inputs for each firm is exactly identical to that of equation (8), and in order to ensure there is no limit pricing, the assumption on the size of each step on the

¹⁶A more detailed exposition of how these price mark-ups can be generated, including Bertrand and Stackelberg versions can be found in Etro (2012).

¹⁷Because incumbents now spend real resources on imitation, the profits paid to households are operational profits π net of research expenditures z^j .

quality ladder in (20) is suitably modified to:

$$(25) \quad \left(\frac{\mu n_t(e)}{\mu n_t(e) - 1} \right)^{\frac{1}{1-\alpha}} < \lambda < \frac{\beta}{\beta - \beta_E} \cdot \frac{1}{1-m}$$

3.2.4. *Research.* Technological progress occurs as described in section (3.1.5) but incumbents are allowed to retain market share by attempting to imitate disruptive technologies. If successful, the incumbent firm stays in the market and competes directly with the entrant. The imitation technology takes on a form that is nearly identical to the innovation technology, with the exception of a scaling parameter ϕ , which is set to a higher value in order to capture the fact that imitating a newly discovered technology is likely to require a substantially lower amount of effort:

$$(26) \quad \text{Prob(incumbent } j \text{ is successful)} = \phi(z_t^j(e))$$

Where $z_t^j(e)q_t$ is total research expenditure on imitation by the j^{th} firm in the e^{th} sector. All of this implies that the value of an existing firm must obey the following dynamic optimisation problem for the incumbent:

$$(27) \quad V_t^j(q_i(e), n_t(e)) = \max_{\{z_t^j(e)\}} \left\{ \pi_t^j(q_t(e), n_t(e)) - z_t^j(e)q_t(e) + \right. \\ \left. + \eta(z_t^E(e))\phi(z_t^j(e))\mathbb{E}_t[\Lambda_{t,t+1}V_{t+1}^j(q_{i+1}(e), n_{t+1}(e))] + \right. \\ \left. + (1 - \eta(z_t^E(e)))\mathbb{E}_t[\Lambda_{t,t+1}V_{t+1}^j(q_i(e), n_{t+1}(e))] \right\}$$

Following line by line, decomposing firm value is relatively simple. The first line is comprised of the firm's operational profits, which finance expenditure in imitation R&D and represent the period flow of income, Σ_t , payable to patient households discussed in previous sections. In the second line, the continuation value of the firm if a function of the probability of both a new innovation occurring and the j -th incumbent successfully copying the new technology. In the event no innovation takes place, the firm retains its incumbency position. The third and final line is simply the firm continuation value in the absence of a successful disruptive innovation, which obviously does not depend on the probability of imitation.

Evidently, this implies that the number of firms in any given sector changes whenever there is a successful discovery: all incumbents initially lose their market share and are replaced by a challenger, which is subsequently joined in production by whichever number

of firms that succeed in adopting the technology. Hence, the law of motion for the total number of firms in each sector is given by the following expression:

$$(28) \quad \mathbb{E}_t(n_{t+1}(e)) = \eta(z_t^E(e)) \left(1 + \int_0^{n_t(e)} \phi(z_t^j(e)) dj \right) + \\ + (1 - \eta(z_t^E(e))) n_t(e)$$

3.2.5. *Resource constraint.* The aggregate resource constraint (3.1.6) is modified to account for the fact that incumbents now engage in research expenditure:

$$Y_t = C_t + C_t^E + I_t + z_t^E + n_t z_j t$$

3.3. Venture capital vs collateralised debt.¹⁸ Allowing entrepreneurs to access financing by selling participation in future firms in exchange for seed money is a substantial deviation from the standard frictions literature, in which borrowing is ordinarily constrained by assuming lenders will be reluctant to part with more than a fraction of the value of an undepreciated asset. The implication is that this contractual relationship more closely resembles that of a venture capital investment. Assuming the finance constraint holds with equality, the total amount returned to the supplier of funds is given by:

$$R_t^b L_t = m_t \eta(z_t^E(e)) \mathbb{E}_t [V_{t+1}^j(q_{i+1}(e), n_{t+1}(e))]$$

And from that we find that the amount earned by entrepreneurs is:

$$(1 - m_t) \eta(z_t^E(e)) \mathbb{E}_t [V_{t+1}^j(q_{i+1}(e), n_{t+1}(e))]$$

Examining the payoffs accrued to each agent allows for a reinterpretation of the economic environment: rather than a restriction on the ease of access to credit, m_t can be thought of as the equity share the venture capitalist is willing to accept for a given amount of seed money $L_t(e)$. In this setting, the loan-to-value ratio (henceforth LTV) common in the financial frictions literature represents the willingness of capitalists to finance new projects. This variable is exogenous to the model and its law of motion is defined as follows:

$$(29) \quad \log(m_t) = (1 - \rho_m) \log(\bar{m}) + \rho_m \log(m_{t-1}) + \epsilon_t^m;$$

¹⁸In sections (4.1) and (5), the two versions of this model will be referred to as Standard RBC CC and Endogenous mark-up CC (EndMkUp CC). This results from changing the way in which entrepreneurs access financing in the two models discussed in sections (3.1) and (3.2).

This approach differs significantly from the standard framework of collateral constraints because it does not restrict capital accumulation, as is the case in those settings. By modifying the entrepreneur's resource constraint in 4, the problem can be restated in a more traditional format:

$$\begin{aligned}
& \max_{\{c_t^E(e), L_t(e), z_t^E(e)\}_{t=0}^{\infty}} U^e = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_e^t \{\log c_t^E(e)\} \\
\text{s.t. } & c_t^E(e) + z_t^E(e)q_i(e) + R_t^b L_t(e) + k_{t+1}(e) = R_t^k k_t(e) + L_t(e) + \eta(z_{t-1}^E(e))V_t^j(q_i(e), n_t(e)) \\
& R_{t+1}^b L_t \leq m_t[(1 - \delta)k_{t+1}(e)]
\end{aligned}$$

By forcing entrepreneurs to post collateral to access financing, restrictions to credit affect both their ability to invest in new capital and engage in research expenditure, which should generate different responses of the model's key variables to exogenous shocks to m_t . An interesting feature of this alternative specification is that it allows the comparison between two sources of financing for entrepreneurs: debt and venture capital. The two versions of the model, with and without endogenous mark-ups, are entirely compatible with this specification of the financing restrictions in the form of a collateral constraint.

3.4. Equilibrium. In addition to assuming a market structure for each sector in which all firms are identical¹⁹, I further restrict the analysis to a fully symmetrical equilibrium in which all sectors are assumed to be identical and at the same level in terms of quality. Additionally, given that most variables grow along the equilibrium path, these are transformed according to $\tilde{x}_t = \frac{x_t}{q_t}$, so that the model and equilibrium conditions only have stationary variables.

3.4.1. Standard model with venture capital financing. The standard Euler and capital accumulation equations are modified to include potential growth rate for the level of

¹⁹In addition to the symmetry for intra-sectoral firms from the assumption of Cournot competition in the model with endogenous mark-ups.

quality²⁰. From the patient household's optimisation problem, we get:

$$(30) \quad 1 = \mathbb{E}_t[\tilde{\Lambda}_{t,t+1}\tilde{R}_{t+1}^b]$$

$$(31) \quad \mathbb{E}_t[\tilde{\Lambda}_{t,t+1}] = \mathbb{E}_t\left[\frac{\beta}{(1 + \tilde{g}_{t,t+1})}\frac{\tilde{c}_t}{\tilde{c}_{t+1}}\frac{v_{t+1}}{v_t}\right]$$

Labour supply can be found from the other first order condition associated with the patient household's optimisation problem:

$$(32) \quad \psi \tilde{h}_t^{-\psi} \tilde{c}_t = \tilde{w}_t v_t$$

From the impatient household's optimal plans, we get the following equation governing the behaviour of research and development spending:

$$(33) \quad -1 + \lambda \frac{d\eta(\tilde{z}_t^E)}{d\tilde{z}_t^j} \mathbb{E}_t\left[(\tilde{\Lambda}_{t,t+1}^E + m_t \tilde{\Omega}_t) \tilde{V}_{t+1}^j(\cdot)\right] = 0$$

Where $\tilde{\Lambda}_{t,t+1}^E$ is the stochastic discount factor for the entrepreneurial household, defined identically to equation (31), and $\tilde{\Omega}_t$ is the Lagrangian multiplier associated with the financing constraint divided by the multiplier associated with the budget constraint. The law of motion for firm value is simply:

$$(34) \quad \tilde{V}_t(\cdot) = \tilde{\pi}_t + (1 - \eta(\tilde{z}_t^E)) \mathbb{E}_t[\tilde{\Lambda}_{t,t+1} \tilde{V}_{t+1}(\cdot)]$$

The Euler equation for the impatient household can be found by solving the first order condition with respect to the amount of seed investment, L_t :

$$(35) \quad 1 = \mathbb{E}_t[(\tilde{\Lambda}_{t,t+1}^E + \tilde{\Omega}_t) \tilde{R}_{t+1}^b]$$

From the solution to the optimisation problem and with both constraints holding with equality, we get the following pair of equilibrium relationships:

$$(36) \quad \tilde{c}_t^E + \tilde{z}_t^E = \tilde{L}_t + (1 - m_{t-1})\eta(\tilde{z}_{t-1}^E) \tilde{V}_t^j(\cdot)$$

²⁰This growth rate is also the balanced growth path growth rate for output, consumption, capital and investment. It is important to note that deviations in this potential growth rate do not necessarily translate into higher output growth; e.g., a temporary shock might increase per capita output in the short run and simultaneously reduce investment in research, thereby reducing the potential growth rate. Once both return to the steady-state level, only permanent changes to R&D expenditure have an effect on the growth rate of per capita output.

$$(37) \quad \tilde{R}_{t+1}^b \tilde{L}_t = \lambda m_t \eta(\tilde{z}_t^E) \mathbb{E}_t[\tilde{V}_{t+1}^j(\cdot)]$$

On the production side, capital accumulation is as follows:

$$(38) \quad \tilde{k}_{t+1}(1 + \tilde{g}_{t+1,t}) = (1 - \delta)\tilde{k}_t + \tilde{i}_t$$

From the intermediate firm's optimal choice of inputs, we can easily derive demand for capital and hours worked, which yields the following equations:

$$(39) \quad \alpha \tilde{y}_t = \mu(\tilde{R}_t - 1 + \delta)\tilde{k}_t$$

$$(40) \quad (1 - \alpha)\tilde{y}_t = \mu \tilde{w}_t \tilde{l}_t$$

Given the assumptions required to determine the equilibrium outcomes, the production function is reduced to:

$$(41) \quad \tilde{y}_t = v_t \tilde{k}_t^\alpha \tilde{l}_t^{1-\alpha}$$

And with a fixed price mark-up over marginal cost, the profit function is simply:

$$(42) \quad \tilde{\pi}_t = \frac{\mu - 1}{\mu} \tilde{y}_t$$

From the law of motion for the quality level, we can derive the equilibrium growth rate of real output along the balanced growth path for this economy:

$$(43) \quad \tilde{g}_{t+1,t} = (\lambda - 1)\eta(\tilde{z}_t^E)$$

The economy's resource constraint is then:

$$(44) \quad \tilde{y}_t = \tilde{c}_t^E + \tilde{c}_t + \tilde{i}_t + \tilde{z}_t$$

Finally, the three laws of motion for the three kinds of shocks are as follows:

$$(45) \quad \log(u_t) = \rho_u \log(u_{t-1}) + \varepsilon_{u,t} \quad \text{Labour productivity shock}$$

$$(46) \quad \log(v_t) = \rho_v \log(v_{t-1}) + \varepsilon_{v,t} \quad \text{Demand shock}$$

$$(47) \quad \log(m_t) = (1 - \rho_m) \log(m) + \rho_m \log(m_{t-1}) + \varepsilon_{m,t} \quad \text{Loan-to-value shock}$$

3.4.2. *Endogenous mark-up with venture capital financing.* With an endogenous market structure, mark-up and profits must be modified to account for the number of firms on the market at any given time, and the equilibrium law of motion for these derived. Additionally, the continuation value for the firm must be modified to take into account the increased probability of remaining in the market, and optimal decisions regarding imitation effort outlined. I discuss each in turn.

The endogenous mark-up follows directly from equation (23) and can be written as:

$$(48) \quad \tilde{p}_t = \frac{\mu \tilde{n}_t}{\mu \tilde{n}_t - 1} \tilde{M}C_t$$

Which implies profits for an individual firm²¹ can be written as:

$$(49) \quad \tilde{\pi}_t^j = \frac{\tilde{y}_t}{\mu(\tilde{n}_t)^2}$$

And the demand for capital and labour inputs adjusted to account for the new endogenous mark-up:

$$(50) \quad \alpha \tilde{y}_t = \frac{\mu \tilde{n}_t}{\mu \tilde{n}_t - 1} (\tilde{R}_t - 1 + \delta) \tilde{k}_t$$

$$(51) \quad (1 - \alpha) \tilde{y}_t = \frac{\mu \tilde{n}_t}{\mu \tilde{n}_t - 1} \tilde{w}_t \tilde{l}_t$$

From the law of motion for the value of the firm, we can derive the optimal amount of imitation effort, which is conditional on the amount of research expenditure by potential entrants.²² Aggregating over all sectors²³:

$$(52) \quad -1 + \lambda \eta(\tilde{z}_t^E) \frac{d\phi(\tilde{z}_t^j)}{d\tilde{z}_t^j} \mathbb{E}_t[\tilde{\Lambda}_{t,t+1} \tilde{V}_{t+1}^j(\tilde{n}_{t+1})] = 0$$

As shown above, the optimal amount of imitation effort by each incumbent is, in equilibrium, related to the research efforts of innovators: a higher fraction of innovative R&D expenditure will mean a higher probability of any given incumbent is replaced, which motivates higher imitation effort from the incumbents.

²¹Sectoral profits are simply the sum of the profits for each individual firm: $y(e) = n(e)y^j(e)$.

²²This, in turn, comes from the incumbent's optimisation problem, which determines the equilibrium value of an intermediate firm.

²³Maintaining the assumption that innovation and imitation probabilities are independent across sectors and of the level of quality in other sectors.

The law of motion for the value of a firm in equilibrium becomes:

$$(53) \quad \tilde{V}_t^j(\tilde{n}_t) = \tilde{\pi}_t^j + (1 - \epsilon_{\phi, z^j})\phi(\tilde{z}_t^j)\eta(\tilde{z}_t^E)\lambda\mathbb{E}_t[\tilde{\Lambda}_{t,t+1}\tilde{V}_{t+1}^j(\tilde{n}_{t+1})] + (1 - \eta(\tilde{z}_t^E))\mathbb{E}_t[\tilde{\Lambda}_{t,t+1}\tilde{V}_{t+1}^j(\tilde{n}_{t+1})]$$

Where ϵ_{ϕ, z^j} is the elasticity of the probability of imitation $\phi(\cdot)$ with respect to level of expenditure on imitation, z_t^j . Knowing optimal innovation and imitation expenditure, we can write the equilibrium expression, or law of motion, for the expected number of firms one period ahead:

$$(54) \quad \mathbb{E}_t[\tilde{n}_{t+1}] = \eta(\tilde{z}_t^E)(1 + \tilde{n}_t\phi(\tilde{z}_t^j)) + (1 - \eta(\tilde{z}_t^E))\tilde{n}_t$$

Aggregate research expenditure is the sum of both innovation and imitation expenditure, where total imitation effort is the sum of the individual efforts of each incumbent:

$$(55) \quad \tilde{z}_t = \tilde{z}_t^E + \tilde{n}_t\tilde{z}_t^j$$

Finally, the aggregate resource constraint becomes:

$$(56) \quad \tilde{y}_t = \tilde{c}_t + \tilde{c}_t^E + \tilde{i}_t + \tilde{z}_t^E + \tilde{n}_t\tilde{z}_t^j$$

3.4.3. Standard model with collateral constraints. Allowing entrepreneurs to accumulate capital and borrow against the value of collateral introduces a few modifications to the equilibrium relationships. In particular, the entrepreneurial household's budget constraint (36) is modified to include capital ownership:

$$(57) \quad \tilde{c}_t^E + \tilde{z}_t^E + \tilde{k}_{t+1}(1 + \tilde{g}_{t,t+1}) = [\tilde{R}_t^k - m_{t-1}(1 - \delta)]\tilde{k}_t + \tilde{L}_t + \eta(\tilde{z}_{t-1}^E)\tilde{V}_t^j(\cdot)$$

And the associated collateral constraint (37) is modified to:

$$(58) \quad \tilde{R}_{t+1}^b\tilde{L}_t = (1 + \tilde{g}_{t,t+1})m_t(1 - \delta)\mathbb{E}_t[\tilde{k}_{t+1}]$$

A final change pertains to the optimal amount of innovation effort, which now captures the fact that while entrepreneurs are constrained, that constraint is not relevant for research efforts as it is affected only by the value of capital in the next period. Therefore, equation (59) becomes:

$$(59) \quad -1 + \lambda \frac{d\eta(\tilde{z}_t^E)}{d\tilde{z}_t^j}\mathbb{E}_t\left[\tilde{\Lambda}_{t,t+1}^E\tilde{V}_{t+1}^j(\cdot)\right] = 0$$

3.4.4. *Endogenous mark-up with collateral constraints.* Introducing collateral constraints in this version of the model produces equilibrium relationships identical to those in section (3.4.3).

4. MODEL SIMULATIONS

In this section I will focus on detailing the various sources for the data used to calibrate the model, as well as outline the basic mechanisms and sources of propagation in the model. One important caveat applies in all discussions of the results: all key variables in the model are transformed to ensure stationarity. Therefore, when discussing the co-movements between research & development and aggregate output, that is equivalent to discussing the relationship displayed by their non-stationary equivalents. On the other hand, when interpreting changes to a single transformed variable, the same cannot be said of the non-stationary equivalent: i.e., a decrease in the transformed real output in the model may not translate into a decrease in actual real output, as it may be associated to an increase in the level of technological advancement.

Nevertheless, the usual interpretations are entirely valid for transformed variables and valid inferences may be drawn from the relationships uncovered by the model.

4.1. Data sources and calibration. The model is calibrated using quarterly data from 1970 to 2008 retrieved from the Bureau of Economic Analysis' National Income and Product Tables. This includes data on gross domestic product, consumption, investment and private R&D expenditure, all deflated using their respective deflators as calculated by the BEA.

To increase comparability between this model and the standard financial frictions framework, some of the parameters are taken from Brzoza-Brzezina et al. (2013). These include the share of capital in equilibrium income, α , set at 0.33; the discount factor for patient households, β , set at 0.995; the equivalent for the impatient households, β_E , set at 0.985; the depreciation rate of capital, δ , set at 0.025; while the inverse Frisch labour supply elasticity, ψ , is set at 1.33²⁴.

The remaining standard parameters, the weight of labour on the utility function, φ , is calibrated in each version of the model to match total aggregate hours worked to 0.4;

²⁴We follow the recommendation of Chetty et al. (2011) to reconcile macro and micro evidence on the empirical estimate for this parameter.

TABLE 1. Structural parameters

Parameters	Values				Description
α	0.33	0.33	0.33	0.33	Capital share
β	0.995	0.995	0.995	0.995	Discount factor patient households
β_E	0.985	0.985	0.985	0.985	Discount factor entrepreneurs
γ	0.071	0.072	0.060	0.060	Research elasticity
δ	0.025	0.025	0.025	0.025	Depreciation rate
η	0.02	0.02	0.019	0.019	Scaling parameter, innovation
ϕ	—	—	0.3	0.3	Scaling parameter, imitation
μ	1.3	1.3	3.431	3.436	Demand elasticity
ψ	1.3(3)	1.3(3)	1.3(3)	1.3(3)	Inverse Frisch elasticity labour
φ	0.145	0.150	0.146	0.151	Weight labour in utility
λ	1.482	1.482	1.482	1.482	Quality jump
m	0.34	0.52	0.34	0.52	Steady-state LTV
ρ_u	0.95	0.95	0.95	0.95	Shock persistence
$\sigma_{u,t}$	0.011	0.011	0.011	0.011	Size of shock
ρ_v	0.947	0.947	0.947	0.947	Shock persistence
$\sigma_{v,t}$	0.01	0.01	0.01	0.01	Size of shock
ρ_m	0.96	0.96	0.96	0.96	Shock persistence
σ_m	0.016	0.016	0.016	0.016	Size of shock
Model	Std RBC VC	Std RBC CC	EndMkUp VC	EndMkUp CC	

the elasticity of substitution between sectors, μ , is set at 1.3 in the two versions of the standard model and at 3.757 in the models with endogenous market structure to match a price mark-up of 1.3²⁵; the scaling parameter associated with the ease of innovation, η , is calibrated in each version of the model to match a steady-state quarterly growth rate of output of 0.007. Calibrating the models to yield an equilibrium price mark-up of 1.3 requires setting the size of the step on the quality level according to the condition in equation (14), which in this context means λ is set at 1.482.

Evidence on the probabilities of innovation across industries are very wide ranging (see Huergo and Jaumandreu (2004), Heffernan et al. (2008) or Hu et al. (2007)) and do not immediately map into the probability of innovation in the model. In equilibrium, this turns out to be 1.45%, which is roughly half of the observed entry rate of new firms of 3.18%²⁶ in US data, according to the Business Dynamics Statistics of the US Census data.

²⁵The value for the mark-up is taken from Jaimovich and Floetotto (2008).

²⁶Annual rate of 12.7%.

The scaling parameter associated with the ease of imitation, ϕ , is set to generate a rate of imitation of 20%, which is within the range of estimates presented in Harabi (1991).

Last among the parameters that are not standard in the financial frictions literature is the elasticity of the probability of innovation and elasticity of the probability of imitation, γ , is calibrated to match private research expenditure to output ratio observed in US data of 1.325%²⁷. The final parameter is the steady-state loan to value ratio, m , which in this setting represents the share of the firm accrued to the venture capitalist is set at 0.34. Empirical evidence on the distribution of pre-IPO shareholdings among investor classes in VC backed enterprises by Barry et al. (1990), suggests the pre-IPO share held by venture capitalists is on average 34%.

TABLE 2. Steady state

Variables	Data	Std RBC VC	Std RBC CC	EndMkUp VC	EndMkUp CC
Research share in output	0.0132	0.0132	0.0132	0.0132	0.0132
Steady state growth rate	0.007	0.007	0.007	0.007	0.007
Endogenous mark-up	1.3	‡	‡	1.3	1.3
Entry rate	0.032	0.015	0.015	0.015	0.015
$R_t^k - R_t^b$ (spread)	N/A	N/A	0.005	N/A	0.005
Capital to loan ratio	2	N/A	2	N/A	2
Impatient HH consumption share	N/A	0.187	0.233	0.183	0.229
Patient HH consumption share	N/A	0.581	0.561	0.585	0.565
Investment share	0.18	0.219	0.193	0.219	0.193
LTV / VC share of firm	0.52/0.34	0.34	0.52	0.34	0.52

‡ Equal to the elasticity of substitution, μ .

As for the exogenous shocks, parameters governing the shock to aggregate labour productivity ($\rho_u = 0.95$, $\sigma_{u,t} = 0.011$) are chosen to match the size of labour productivity shocks in US data from the BLS labour productivity series, the parameters for the LTV ratio ($\rho_m = 0.96$, $\sigma_{m,t} = 0.016$) are taken from Brzoza-Brzezina et al. (2013). The parametrisation of the aggregate demand or preference shock is ($\rho_v = 0.95$, $\sigma_{d,t} = 0.01$).

²⁷Evidence presented in Comin and Gertler (2006) and Griliches (1991) indicates that an elasticity of new patents with respect to expenditure in research and development of 0.5 is a reasonable estimate, but other authors, like Barrau (2008), place this elasticity at 0.077 to match certain features of the data, and the values chosen here are of the same order of magnitude. This discrepancy between empirical estimates and calibrated values suggests that a mechanism that separates firm creation from successful innovations might better reconcile these facts.

A full list of values for the parameter values chosen for these simulations is included in the parameter table [1].

5. DISCUSSION

The model broadly replicates the empirical features of research spending in the data, namely the pronounced pro-cyclicality of level expenditures and the strong counter-cyclicality of the ratios of R&D with respect to both output and investment spending, which is entirely consistent with the empirically observed research expenditure "smoothing" in the data both at aggregate and disaggregated levels. Looking at the first three lines of table (3) provides a brief summary of the main features of the four models: they all predict positive correlations between deviations in output and deviations in R&D spending and negative correlations between the aforementioned ratios and economic activity.

TABLE 3. Calibration: correlations with output

	Data	Std RBC VC	Std RBC CC	EndMkUp VC	EndMkUp CC
(dy_t, dz_t)	0.452	0.772	0.733	0.765	0.756
$(dy_t, d\frac{z_t}{y_t})$	-0.249	-0.756	-0.549	-0.762	-0.118
$(dy_t, d\frac{z_t}{z_t+i_t})$	-0.828	-0.858	-0.743	-0.872	-0.779
(dy_t, dL)	0.486	N/A	0.041	N/A	0.043
(dy_t, dc)	0.873	0.633	0.282	0.639	0.275
(dy_t, dk)	0.163	0.299	0.419	0.298	0.419
(dy_t, dg_t)	0.286	0.770	0.733	0.738	0.690
(dy_t, dw_t)	0.848	0.957	0.765	0.955	0.765
(dy_t, dr_t)	0.682	0.911	0.393	0.912	0.394
Model specific outcomes					
(dy_t, dz_t^E)	N/A	N/A	N/A	0.740	0.688
(dy_t, dz_t^j)	N/A	N/A	N/A	0.813	0.223
$(dy_t, d\frac{z_t}{y_t})$	N/A	N/A	N/A	-0.794	-0.488

The model that incorporates endogenous mark-ups and collateral constraints comes closest to replicating the response of R&D expenditure to output in the data, though that gain comes at the expense of a poor performance in terms of matching the correlation between loans (L_t) and output or the correlation between consumption and output. The co-movement between loans and output in the literature (see Brzoza-Brzezina et al. (2013)) has the opposite sign to that observed in the data and is a well known, and

undesirable, feature of collateral constraint specifications in the mould of Kiyotaki and Moore (1997), suggesting that alternative specifications for financial frictions might better capture the correlation between lending and economic activity. In the simulations presented throughout, this correlation takes on the right sign but is much smaller than its empirically observed counterpart. Another striking feature of the models with collateral constraints is the poor performance in terms of matching the correlation between output and consumption, which is much lower than correlations in the data.

Despite all models making similar, though quantitatively different, predictions about how R&D correlates with output along the cycle, the structure of the more complete models allows us to examine the response of all the different components of research expenditure to the three different sources of shocks. Table (4) summarises these results.

TABLE 4. Calibration: correlations with output by source of shock

Standard RBC VC			Standard RBC CC		
Technology	Demand	LTV	Technology	Demand	LTV
(dy_t, dz_t)	0.945	-0.605	0.611	0.978	-0.988
$(dy_t, d\frac{z_t}{y_t})$	-0.848	-0.835	0.581	-0.829	-0.995
$(dy_t, d\frac{z_t}{z_t+i_t})$	-0.837	0.998	0.690	-0.832	-0.983
Endogenous mark-up VC			Endogenous mark-up CC		
Technology	Demand	LTV	Technology	Demand	LTV
(dy_t, dz_t)	0.941	-0.694	0.617	0.981	-0.988
(dy_t, dz_t^E)	0.941	-0.692	0.636	0.978	-0.988
(dy_t, dz_t^j)	0.940	-0.699	-0.385	0.989	-0.990
$(dy_t, d\frac{z_t^j}{y_t^j})$	-0.802	-0.858	-0.623	-0.760	-0.996
$(dy_t, d\frac{z_t}{y_t})$	-0.838	-0.861	0.582	-0.838	-0.996
$(dy_t, d\frac{z_t}{z_t+i_t})$	-0.835	1.000	0.698	-0.835	-0.986

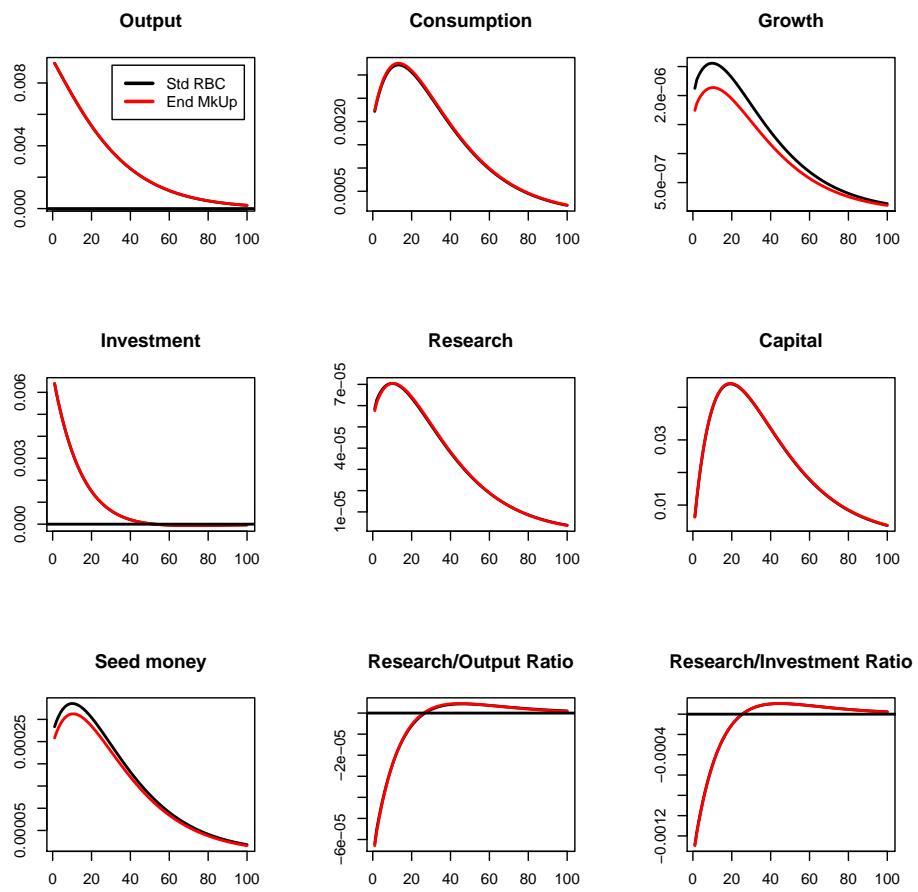
In order to facilitate the presentation of the results, section (5.1) looks at the performance of the models where entrepreneurs relinquish a fraction of equity in exchange for seed money investments, and section (5.2) looks at the models in which impatient households face a collateral constraint when borrowing to finance consumption and research expenditure.

5.1. Venture capital financing.

5.1.1. *Productivity shock.* A temporary increase in aggregate productivity leads to standard responses from all the components of expenditure: consumption by both types of

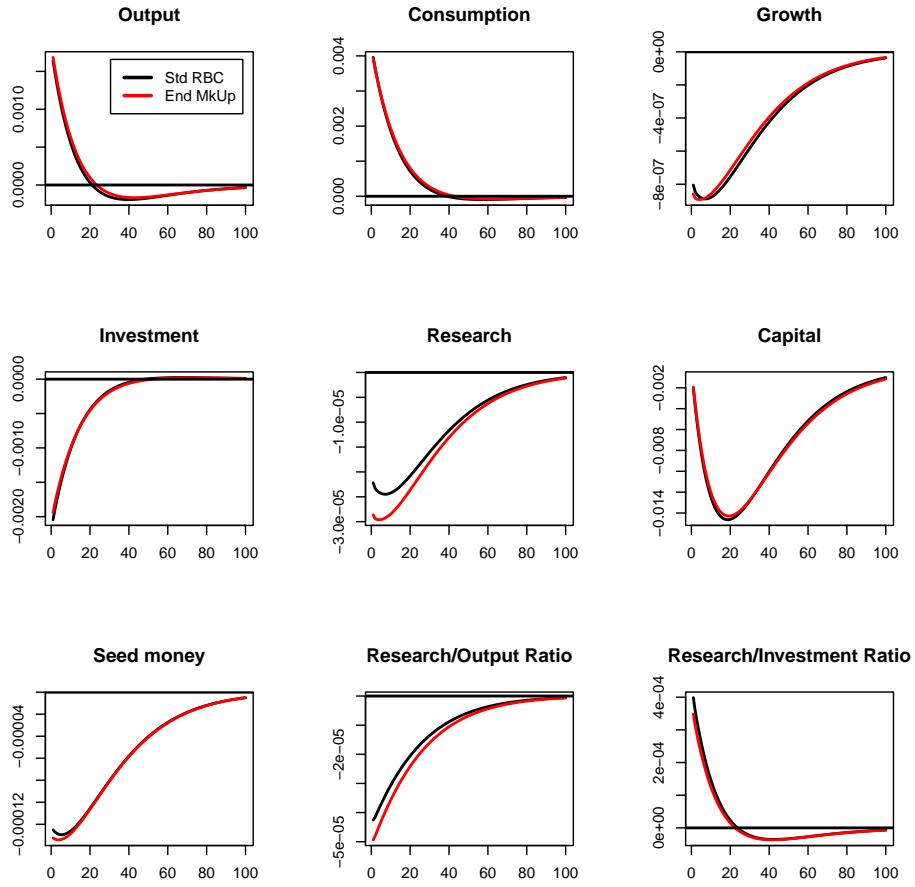
household goes up, as does investment in physical capital and research given the increased returns to investing in capital and to investing in an additional unit of R&D. Figure (2) shows the impulse response functions for the models with and without endogenous mark-ups, with the pro-cyclicality of R&D, as well as the counter-cyclicality of the ratios of research spending to output and investment, readily apparent. The three first columns of table (4) summarise the correlations between the various measures of research spending and output: expenditure increases in both models along with output while the correlation between the ratios and output is negative.

FIGURE 2. Impulse response functions for productivity shock in models with venture capital



From it, it is also clear that investment in research by both entrepreneurs and established firms moves in the same direction: positive productivity shocks lead to more innovation and increased imitation efforts, which results in highly pro-cyclical aggregate research expenditure.

FIGURE 3. Impulse response functions for demand shock in models with venture capital



5.1.2. *Aggregate demand shock.* Aggregate demand shocks are modelled as 'preference' shifters, in that they temporarily increase the relative utility of consumption and therefore increase the supply of labour which, in turn, increases the level of economic activity, as can be seen in figure (3). Productivity does not increase, however, and patient households therefore privilege consumption at the expense of productive economic activity, lowering

investment both in physical capital and in the amounts made available to budding entrepreneurs. With less resources available for innovative activity, research expenditure decreases along with investment in physical capital. Financial frictions in the form of a need for entrepreneurs to finance expenditure by requiring investment from the patient households amplify the counter-cyclical effect of these on research spending. Predictably, the ratio of R&D to output also falls sharply, while the its ratio with respect to total investment increases as investment in physical capital is more affected by patient household's reduced willingness to invest.

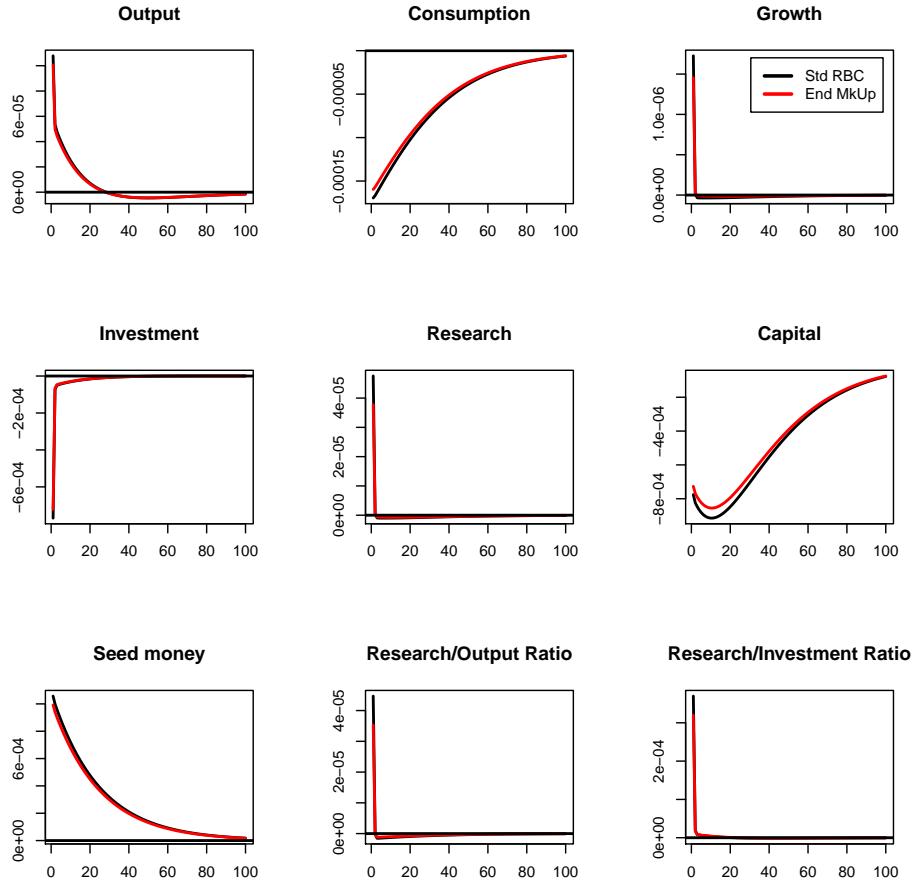
The Schumpeterian 'opportunity cost' hypothesis is offered a brief lifeline here: under certain conditions, an increase in economic activity could potentially generate a counter-cyclical response from R&D spending. Specifically, and contrary to what happens in the case of a productivity shock, demand shocks siphon resources away from investments that have future payoffs towards consumption, which is precisely what the opportunity cost hypothesis suggests would occur: increases to demand for a product that are not based on productivity increases lead firms to prioritise production over expenditure in R&D (as can be seen in table (4) in the correlation between output and z_t^j) so as to increase the amount of profits paid to households. Despite firm valuations increasing, increased consumption by patient households reduces the amount of resources available to entrepreneurs and, consequently, consumption by this latter class of agents falls. This result does not hinge on the existence of constraints, but these amplify the mechanism by making it harder for impatient households to access finance.

Finally the response of research spending over total investment to a demand shock is strongly pro pro-cyclical, which sharply and distinctively contrasts with both its response to a productivity shock and available empirical evidence. This, however, is not surprising: because of R&D smoothing on the part of both firms and entrepreneurs, investment in physical capital decreases by much more than the decrease in research expenditure. Therefore, the ratio reported on in table (4) will always respond pro-cyclically given the large impact on investment expenditure.

5.1.3. *Loan to value shock.* The final source of fluctuations analysed takes the form of an exogenous variation in the loan-to-value ratio m_t . An increase in this value means the constraint on either the amount impatient households are willing to invest outright in a new venture is relaxed. Absent other frictions like Calvo pricing or adjustment costs

to investment in physical capital, the effect of a shock to the availability of financing on investment and/or credit is reversed as soon as the shock is realised²⁸. Productivity or

FIGURE 4. Impulse response functions for LTV shock in models with venture capital



demand shocks have longer lasting effects on expenditure variables because in either case the effect on productivity or preference for consumption have a high degree of persistence and, therefore, optimal behaviour implies that most variables take on a similar degree of autocorrelation. A shock to the loan-to-value ratio has a different impact because as it loosens or tightens the constraint on financing, it does so by either making more or less

²⁸It is not clear whether this would have significant effects even in the presence of the final good and capital production frictions common in New Keynesian models. See Brzoza-Brzezina et al. (2013).

funds available for entrepreneurs to invest in research or capital spending. This means the total cost of repayment in the following period is respectively higher or lower, to which the optimal response of households is to bring expenditure to near steady-state levels. The initial impact on investment, however, means capital is lower for the duration of the shock, reducing productive capacity and leading to a slower adjustment of production and consumption.

This does not allow us to examine what happens to the main macroeconomic variables for the duration of the shock, but the direction of the impact is clearly discernible: when venture capital is the source of financing for impatient households, research expenditure increases and investment in physical capital falls. Due to the temporary decrease in investment, the physical capital stock decreases but production is still above its balanced growth path levels because of increased labour usage, driven by an increase in expenditure from higher research spending and higher consumption by entrepreneurs.

Table (4) outlines the strongly procyclical behaviour of aggregate research expenditure in both variants of the model with, surprisingly, the ratio of R&D to output and its ratio to total investment displaying equally pro-cyclical behaviour. This occurs because rather than financial frictions amplifying a shock originating elsewhere in the economy, when the shock originates in the financial sector, research spending drives the increase in output, rather than the reverse occurring. An interesting prediction of the model with endogenous mark-ups, however, is that level of research spending by firms, rather than entrepreneurs, will behave counter-cyclically. The implication then is that easier access to financing will drive the expenditure levels of constrained agents upwards, while other agents will optimally choose to reduce their expenditure.

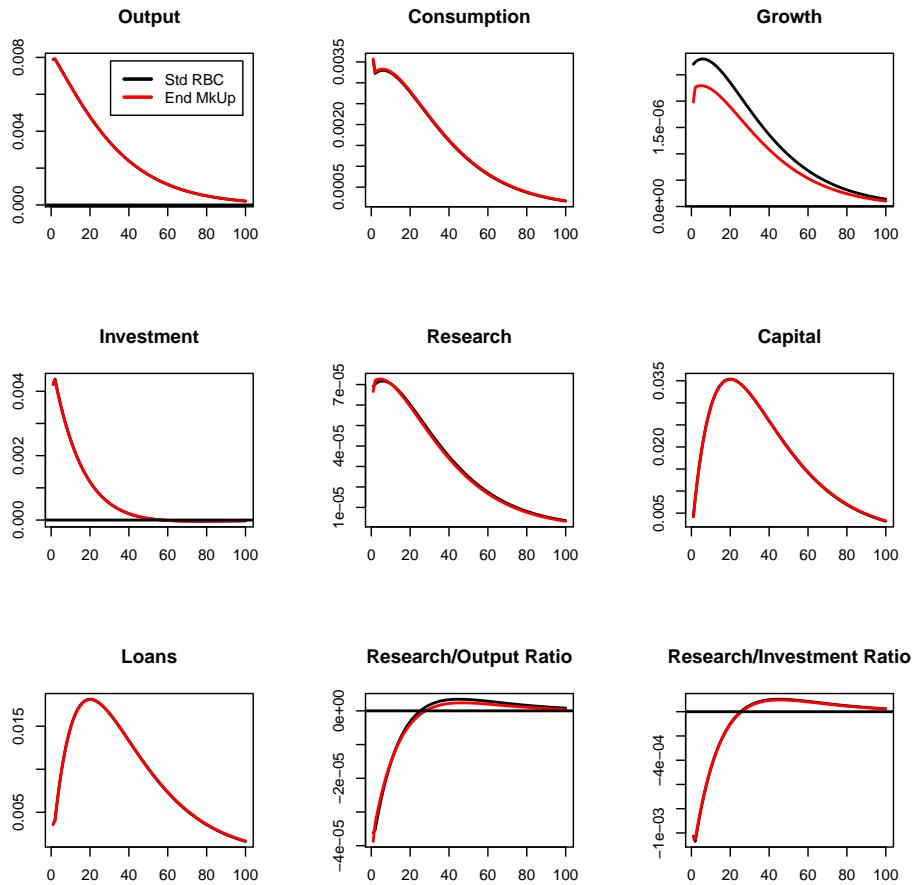
While the literature on the impact of financial constraints on entrepreneurial activity often focuses on their role as amplification mechanisms, by modelling these constraints directly a few distinctive implications can be drawn by examining their impact on expenditure when shocks originate in the financial linkages between economic agents. The model presented here suggests that these may contribute to the pro-cyclical behaviour of R&D spending exactly in the manner suggested by earlier work (see Aghion et al. (2012))²⁹.

²⁹An important *caveat* here relates to the choice of variable by these agents. As I have argued, the ratio of research expenditure to output and investment almost always displays counter-cyclical behaviour and its response to fluctuations should therefore be interpreted with some caution.

5.2. Collateral constraints.

5.2.1. *Productivity shock.* The response of these variables to a positive technological shock in the two variants of the model with collateral constraints is identical to that of the previously examined cases: expenditure goes up while the ratio of R&D to output falls.

FIGURE 5. Impulse response functions for productivity shock in models with collateral constraints



This can also very clearly be seen in the correlations table (4), which shows the correlations between all measures of research spending and output take on the same signs and very similar values regardless of the version of the model being analysed. More succinctly, productivity shocks elicit pro-cyclical responses from research expenditure at all levels of aggregation, with both entrepreneurs and established firms responding identically to this

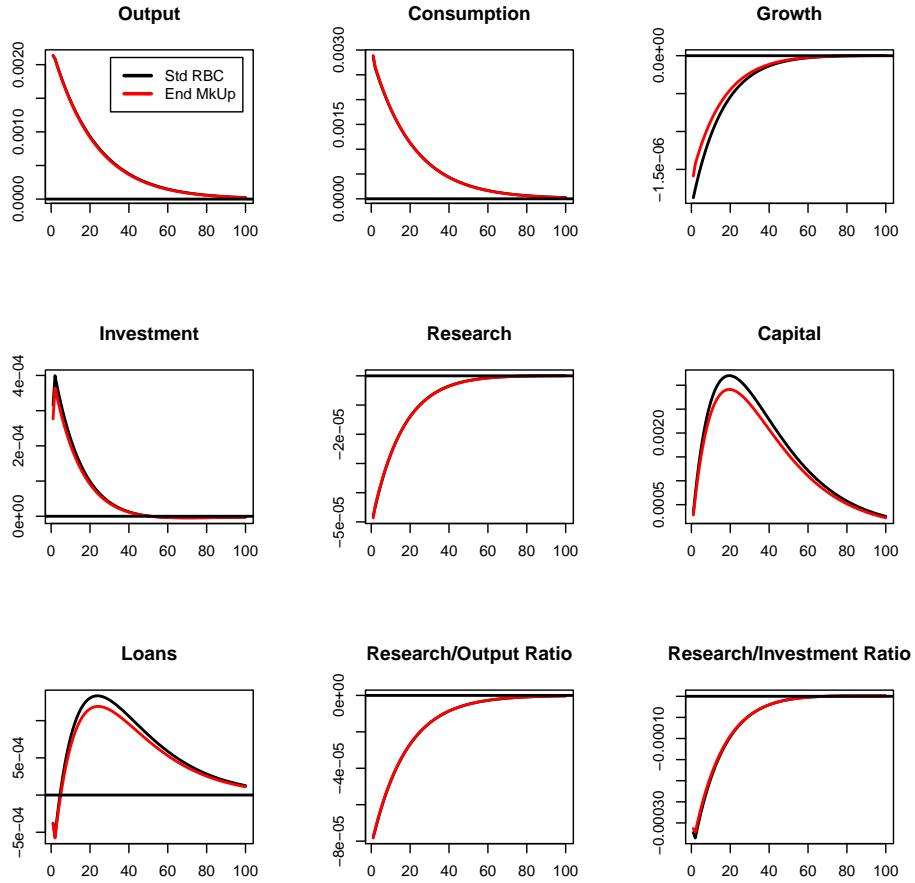
type of exogenous shock. Furthermore, because of R&D smoothing, research expenditure is less volatile than either output or investment expenditure, implying that its ratio with respect to either of these behaves counter-cyclically. Given that productivity shocks are generally identified in business cycle models as driving a substantial fraction of the overall variance for key macroeconomic aggregates, this would suggest that the behaviour of the variables of interest in the model broadly corresponds to empirical observation. In other words, if the observed empirical variation in production is driven by variations in productivity rather than changes in demand, the model offers theoretical predictions that precisely align with empirical findings.

5.2.2. Demand shock. In sharp contrast with the response to a productivity shock, measures of the cyclical response of R&D to demand driven shocks point to a strongly counter-cyclical pattern, regardless of the manner in which impatient households finance their expenditure. The only exception to this pattern is the behaviour of the ratio of aggregate research spending to total investment. When entrepreneurs finance innovative activity by alienating future equity, patient households will choose to reduce investment in physical capital which, being more volatile than R&D spending, decreases substantially more, ensuring that the ratio behaves pro-cyclically. Here, however, because it is entrepreneurs that decide optimal levels of investment and because these agents can only borrow up to a fraction of the future value of their posted collateral, capital spending increases as research spending falls. This ensures that the ratio of R&D to total investment will fall, behaving counter-cyclically.

These results suggest that if agents can correctly identify the source of the shock³⁰, the optimal response to temporary shifts in consumers' preference for consumption is to devote less resources to both investment in physical capital and research expenditure, reflecting the increased returns to generating output to meet the increase in consumption. In the case where entrepreneurs are constrained in their optimal choice of capital expenditure, however, the increase in demand today drives down the borrowing rate and relaxes that constraint, leading to higher investment spending in response. The response of research expenditure is, however, invariably counter-cyclical across model specifications.

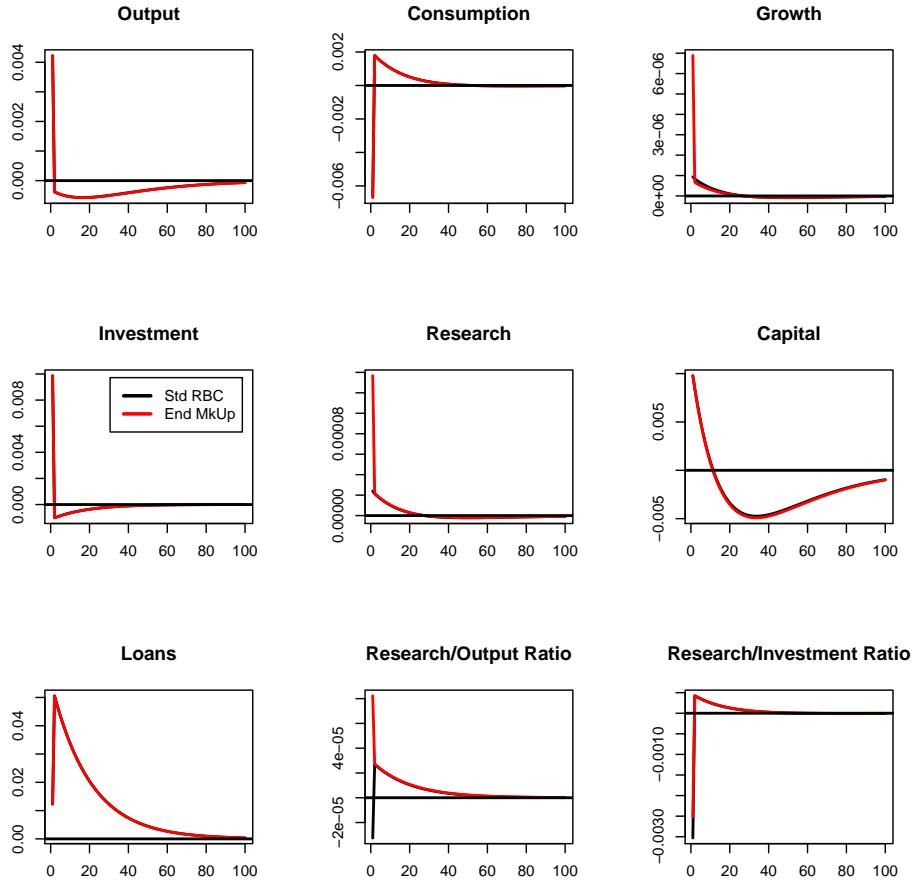
³⁰Which in this context is a consequence of the assumption of a rational expectations equilibrium with full information.

FIGURE 6. Impulse response functions for demand shock in models with collateral constraints



5.2.3. *Loan-to-value shock.* As mentioned in the discussion of the response of research spending to loan-to-value shocks when entrepreneurs resort to equity financing, frictions in the financial linkages between both types of household can act as an amplification mechanism for shocks originating elsewhere. Its response to shocks originating in these linkages suggested how they may not only contribute but also be responsible for a more pronounced pro-cyclical pattern of R&D. Because investment spending decreased in that case, the measured elasticity of research spending to output would exceed one, meaning a higher volatility of R&D than output and measured pro-cyclical behaviour in all measures of research spending, including its ratios to output and total investment. It also highlighted the counter-cyclical response of imitation efforts by established incumbents.

FIGURE 7. Impulse response functions for LTV shock in models with collateral constraints



However, when faced with borrowing constraints, this measured response changes significantly. Because they predict different responses of research spending, I look at the models with and without endogenous mark-ups separately. In the exogenous mark-up version, the measured pro-cyclicality of aggregate R&D is modest and much closer to zero than in any other model, suggesting a very moderate response of research spending to a relaxing of the credit constraint. Hence, the ratio of research spending to output and investment behave counter-cyclically given that R&D responds much less to a tightening of the borrowing constraint than either output or capital investment. In the endogenous mark-up version, the response of research spending is much more pronounced, as can be seen in the middle graph of figure (7), and this drives the observed pro-cyclicality of the

ratio of R&D to output: the impact on research spending is higher than the impact on output. Contrary to what happens in the venture capital financing models, because the constraint directly affects capital investment, its relaxing leads to an increase in investment spending.

6. CONCLUSION

In attempting to reconcile the observed pro-cyclicality of R&D in the data with the Schumpeterian hypothesis that a pro-cyclical opportunity cost of doing research would lead to a counter-cyclical pattern, it has long been observed that different sources of fluctuations might generate a different response of research spending. The results of this exercise suggest that aggregate productivity shocks are an important explanation for the observed positive correlation between output and research activity: changes in productivity drive investment in both innovation and physical capital and therefore generate pro-cyclical responses in expenditure variables - a tide that raises all boats. Financial frictions amplify this mechanism because lending and venture capital seed money respond pro-cyclically to temporary productivity shocks and, therefore, firms and sectors in which these constraints are more significant can be expected to have more pro-cyclical R&D spending. A weaker form of the opportunity cost hypothesis still holds, however, if we look at the ratio of research expenditure to output and with respect to the sum of R&D and physical capital investment. This form of R&D 'smoothing' suggests that the trade-off is still relevant as it forces firms and entrepreneurs to invest proportionally less in research than they do in physical capital.

Demand shocks, in the form of preference shifts towards consumption, generate counter-cyclical responses from expenditure variables. This is the result of households shifting resources away from all forms of investment towards consumption at the same time that wages fall and employment increase. This comes closest to the explanation originally proposed by Schumpeter: temporary changes to demand that put pressure on output will lead to counter-cyclical responses of research expenditure. The ratio of R&D to investment behaves pro-cyclically when patient households are the capital owners (as a result of more resources being diverted away from investment) and counter-cyclically when entrepreneurs are the capitalists, which is due to the spread between the rate of return on capital and the borrowing rate behaving counter-cyclically.

Finally, shocks originating directly in the financial linkage between patient and impatient households generate pro-cyclical behaviour in research expenditure from two main channels. In the venture capital financing model, the spread between the internal return rate on a new innovation and the borrowing rate is counter-cyclical, which means borrowing and investment in R&D behave pro-cyclically. In the collateral constraint model, a

counter-cyclical spread again means pro-cyclical lending and pro-cyclical investment, this time in physical capital. Research expenditure will then track the cyclical behaviour of output and investment.

To the extent that productivity shocks dominate other sources of fluctuations as explanations for the volatility of output, the model's predictions are validated by the empirically observed R&D smoothing. It also suggests that empirical research should try to clearly differentiate changes in output that originate in productivity improvements from those that are simple fluctuations in aggregate demand, as R&D spending may behave differently to each type of shocks.

Furthermore, the commonly accepted view that financial constraints might be the reason why research spending tracks the cycle is only partially validated by the predictions of the model or the empirical results discussed in the previous chapter. Indeed, financial frictions amplify the effect of fluctuations on research spending, which means that while it will always behave pro-cyclically in response to technology shocks and counter-cyclically in response to demand shocks; their presence will generate more pronounced responses but it is not the central driving force for the cyclical behaviour of research spending and without them a similar pattern would be observed. Hence, they are not a sufficient explanation for why R&D may be pro-cyclical. On the other hand, when fluctuations originate in the financial sector, they tend to generate pro-cyclical responses in research spending. To the extent that these contribute to a significant fraction of the fluctuations in measured output, they would in principle add to the measured pro-cyclical of research expenditure in the absence of a precise identification strategy that could isolate the source of the fluctuations.

Ignoring key features of the New Keynesian literature such as frictions to price setting behaviour or investment adjustment costs significantly impairs the model's ability to fully exploit the effects of fluctuations originating in the financial sector, and is a promising avenue for future research. As noted, while on a conceptual level collateral constraints of the type proposed by Kiyotaki and Moore (1997) seem to provide a natural strategy to incorporate financial constraints in a model of this nature, the relatively inferior performance of this model to alternative specifications suggests future work may explore whether these would point to substantially different conclusions.

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